AN EXAMINATION OF THE WALD STOPPING BOUNDS FOR THE SEQUENTIAL PROBABILITY RATIO TEST

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THESIS

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Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN OPERATIONS RESEARCH

from the

NAVAL POSTGRADUATE SCHOOL
April 1970

ABSTRACT

An examination of the Wald stopping bounds for the Sequential Probability Ratio Test (SPRT) is made by comparing results obtained from Monte Carlo simulations of sequential sampling tests with results obtained using Wald formulations. Operating Characteristic, ASN, and V[N] values are presented for tests sampling from each of eight Binomial, 14 Exponential, and 24 Normal distributions. An extensive bibliography of references associated with SPRT is included.

TABLE OF CONTENTS

LIST OF TABLES

NORMAL	0.0	1.0	.010	.001	----	62
NORMAL	0.0	1.0	.010	.010	----	63
NORMAL	0.0	1.0	.010	.100	----	64
NORMAL	0.0	1.0	.010	.200	----	65
NORMAL	0.0	1.0	.100	.001	----	66
NORMAL	0.0	1.0	.100	.010	----	67
NORMAL	0.0	1.0	.100	.100	----	68
NORMAL	0.0	1.0	.100	.200	----	69
NORMAL	0.0	2.0	.010	.010	----	70
NORMAL	0.0	2.0	.010	.100	----	71
NORMAL	0.0	2.0	.100	.010	----	72
NORMAL	0.0	2.0	.100	.100	----	73
NORMAL	0.0	3.0	.010	.010	----	74
NORMAL	0.0	3.0	.010	.100	----	75
NORMAL	0.0	3.0	.100	.010	----	76
NORMAL	0.0	3.0	.100	.100	----	77

Note: A variance of 1.0 was used with all Normal Distributions.

PREFACE AND ACKNOWLEDGEMENTS

The terms "approximate" and "exact" used in this thesis are associated with the Wald derivation of the SPRT and the author's Monte Carlo simulation model of a SPRT procedure, respectively. These terms are used only to distinguish between the two sources of results.

The computer programs were tested at the Naval Postgraduate School's Computer Center in the period from March 1969 to January 1970. All references in this thesis to times and other computer items are related to this IBM-360 computer.

I wish to express my sincere appreciation to Professor Donald R. Barr, to the personnel at the library of the Naval Postgraduate School, and to the computer facility and its staff.

I. OBJECTIVE

In this thesis the accuracy of the Wald approximate decision limits of the Sequential Probability Ratio Test (SPRT) are investigated. The Wald approximate limits are compared with "exact" emperical limits obtained by Monte Carlo simulation. An extensive bibliography of references associated with the SPRT is included.

II. HISTORY OF SPRT

In statistical theory the size of a sample may or may not be fixed prior to observation of certain sample values. If, in a test of hypotheses, the sample size is not fixed in advance, the decision to terminate sampling may depend upon the values of the previous samples. Such a test is said to be sequential.

The first mention of sequential test procedures was by H. F. Dodge and H. G. Romig who, in 1929, constructed a Double Sampling Plan. Prior to World War II, there are not many entries in the literature concerning sequential procedures. During World War II the Statistical Research Group of Columbia University operated under a contract with the Office of Scientific Research and Developement and was directed by the Applied Mathematics Panel of the National Defense Research Committee. Milton Friedman and W. Allen Wallis, members of the research group, recognized the great potentialities and far reaching consequences that sequential analysis might have; consequently various members of the group, and in particular A. Wald, worked out what is known as the SPRT. In the early 1940's many of these results were classified, however, the Restricted classification was removed in 1945.

Abraham Wald, in 1943, worked out the basic principles for the SPRT [Wald, 1943]. During the next two years Wald continued working on the basic principles of the SPRT, including a general consideration of cumulative sums of independent random variables which gives the Operating Characteristic (OC) curve of any SPRT, and the characteristic function of the number of observations required by the test [Wald, 1944].

During this same period of time, independent work on sequential inferences was also conducted in England by G. A. Barnard [Barnard, 1946] who derived general results similar to those obtained by the Statistical Research Group at Columbia.

Since the unfortunate death of A. Wald in 1950, there have been many varied contributions to the literature of sequential methods. These contributions tend to deal primarily with specific families of distributions. Many of the articles are listed in the bibliography.

III. THEORETICAL BACKGROUND OF THE SPRT

The Sequential Probability Ratio Test of a simple Null Hypothesis against a simple Alternate Hypothesis differs from fixed sample size hypothesis tests in that it is conducted in stages, where a stage constitutes evaluation of an observation. At each stage one of three alternatives is chosen: (1) discontinue sampling, accept the null hypothesis; (2) discontinue sampling, reject the null hypothesis; (3) draw another observation. The procedure continues until one of the alternatives (1) or (2) is chosen. Under quite general assumptions, the probability of eventual termination of the SPRT is equal to one.[1]

A. DEFINITION OF SPRT

Let the distribution of the random variable, X, under consideration be given by the density or mass function, $f(x;\, \theta)$. Let H_o be the Null Hypothesis that $\theta = \theta_o$ and H_1 be the Alternate Hypothesis that $\theta = \theta_1 > \theta_o$. Therefore the density or mass function of X will be $f(x;\, \theta_o)$ when H_o is true and $f(x;\, \theta_1)$ when H_1 is true. Successive independent observations on X will be denoted x_i, $i = 1,2,\ldots$. The SPRT is based on the likelihood ratio,

$$\Pi_n = \prod_{i=1}^{n} \frac{f(x_i;\, \theta_1)}{f(x_i;\, \theta_o)}$$

[1] A. Wald, Sequential Analysis, Wiley, N.Y., 1947, p.157.

and two positive numbers A and B, $A > 1$ and $B < 1$. After each observation on X, the procedure for choosing one of the three alternatives is:

1) if $\Pi_n \leq B$, Discontinue Sampling, Accept H_o
2) if $\Pi_n \geq A$, Discontinue Sampling, Accept H_1
3) if $B < \Pi_n < A$, Draw another observation.

B. DERIVATION OF STOPPING BOUNDS

The two constants A and B are determined so that the test will have (nearly) the prescribed probabilities, α and β, of making errors, where α is the probability of making a Type I error (Rejecting H_o when it is true) and β is the probability of making a Type II error (Accepting H_o when it is false). Exact values for A and B could, in principle, be obtained from the following equations given in the first that H_o is true and in the second that H_1 is true:

$$\alpha = P[\Pi_1 \geq A] + P[\Pi_2 \geq A,\ B < \Pi_1 < A] + \ldots\ldots$$

$$\beta = P[\Pi_1 \leq B] + P[\Pi_2 \leq B,\ B < \Pi_1 < A] + \ldots\ldots \quad .$$

In practice, approximations for A and B developed by Wald are usually used, where α and β are specified apoiri.[2] The Wald approximate stopping bounds are

$$A = \frac{1 - \beta}{\alpha}$$

$$B = \frac{\beta}{1 - \alpha} \quad .$$

---

[2] A. Wald, Sequential Analysis, op.cit., pp. 40-44.

The boundaries used in the tests performed for this thesis were formulated as two diagonal lines with proper intercepts.[3] These boundaries are called the acceptance number, A_n, and the rejection number, R_n. These numbers are obtained by setting the logarithm of Π_n, the likelihood ratio, equal to the logarithms of A and B. The resulting test is the Wald SPRT with stopping bounds A and B.

C. RANDOM WALK OF THE OBSERVATION RESULTS

At each observation the value of the test statistic was tabulated. The stepwise values of the test statistics can be graphed with the abscissa being the number, n, of observations made and the ordinate being the value of the test statistic. The boundaries, A_n and R_n, will limit the steps of the random walk. The test terminates when either boundary is reached or surpassed by a value of the test statistic.

D. THE OC FUNCTION OF THE SPRT

The Operating Characteristic (OC) Function, $L(\theta)$, is defined as the probability that the sequential test will lead to acceptance of H_o when θ is the true value of the parameter. Using the approximations on the stopping

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[3] A. Wald, "Sequential Tests of Statistical Hypothesis," Annals of Math. Stat., Vol. 16, No. 2, 1945, pp. 160-62.

bounds mentioned above, Wald showed[4] that the OC function could be approximated by

$$L(\theta) \doteq \frac{A^{h(\theta)} - 1}{A^{h(\theta)} - B^{h(\theta)}} \qquad (1)$$

where $h(\theta)$ is a non-negative real number such that

$$\int_{-\infty}^{\infty} \left[\frac{f(x;\ \theta_1)}{f(x;\ \theta_o)}\right]^{h(\theta)} f(x;\ \theta)dx = 1 \quad , \qquad (2)$$

or the equivalent summation in the case f is a mass function.

It is sometimes difficult in practice to obtain the value for $h(\theta)$ for each of various given values of θ, in such cases a "reverse" process may be used: set $h(\theta)$ equal to a non-zero real number and compute a corresponding value of θ. This technique was used in computing $L(\theta)$ for the Binomial and Exponential distributions in this thesis.

E. THE EXPECTED SAMPLE SIZE OF THE SPRT

As mentioned above, with probability one the SPRT eventually terminates. Thus, using the approximate boundaries, A and B, and disregarding the "excess" of Π_n over these boundaries at termination,

$$P[\text{Ln } B = Z \mid \theta] \doteq L(\theta)$$

$$P[\text{Ln } A = Z \mid \theta] \doteq 1 - L(\theta)$$

---

[4] A. Wald, <u>Sequential Analysis</u>, op.cit., pp. 48-52, 161-64.

where $Z = \sum_{i=1}^{N} \mathrm{Ln}\, \Pi_i = \sum_{i=1}^{N} Z_i$, and where in turn N is the sample size required for termination. Therefore, the conditional expected value of Z, given θ, is approximately

$$E[Z \mid \theta] \doteq [1-L(\theta)]\,\mathrm{Ln}\,A + L(\theta)\mathrm{Ln}\,B \quad .$$

Utilizing Wald's Fundamental Identity,[5] the conditional expected value of Z, given θ, can be written

$$E[Z \mid \theta] = E[\sum_{i=1}^{N} Z_i \mid \theta] = [E]N \quad E[Z_i \mid \theta] \quad .$$

The Expected Sample Size for the test is therefore given approximately by

$$E[N] = \frac{E[Z \mid \theta]}{E[Z_i \mid \theta]} \doteq \frac{[1-L(\theta)]\mathrm{Ln}\ A + L(\theta)\mathrm{Ln}\ B}{E[Z_i \mid \theta]} \quad . \qquad (3)$$

If $E[Z_i \mid \theta] = 0$, one may approximate the Expected Sample Size by

$$E[N] = \frac{E[Z^2 \mid \theta]}{E[Z_i^{\,2} \mid \theta]} \quad .$$ [6]

F. VARIANCE OF SAMPLE SIZE OF THE SPRT.

It would appear that the Variance of the Sample Size, (N), could be approximated in certain cases using an

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[5] A. Wald, Sequential Analysis, op.cit., pp. 159-60.

[6] A. Wald, "Differentiation under the Expectation Sign in the Fundamental Identity of Sequential Analysis," Annals of Math. Stat., Vol. 17, No. 4, p. 472, December 1946.

approach similar to that for $E[N]$. During his literature review the author encountered only three references to such an approximation [Wald, 1945a], [Walker, 1950], [Cox and Roseberry, 1966a].

Since

$$V[N] = E[N^2] - E^2[N] \tag{4}$$

an approximation for $V[N]$ may be developed as follows: $E^2[N]$ can be approximated using Eq. (3). An approximate value for the $E[N^2]$ may be derived from

$$E[Z^2 | \theta] = E[(\sum_{i=1}^{N} Z_i)^2 | \theta] = E[\sum_{i=1}^{N} Z_i^2 + \sum_{\substack{i=1 \\ i \neq j}}^{N} Z_i Z_j] \quad .$$

Utilizing Wald's Fundamental Identity, this may be written

$$E[Z^2 | \theta] = E[N]E[Z_i^2 | \theta] + E[N(N-1)]E^2[Z_i | \theta]$$

where, as before, Z_i and Z_j are independent and identically distributed. Expanding and collecting terms

$$E[Z^2 | \theta] = E[N](E[Z_i^2 | \theta] - E^2[Z_i | \theta]) + E[N^2]E^2[Z_i | \theta]$$

$$E[Z^2 | \theta] = E[N]\ V[Z_i | \theta] + E[N^2]\ E^2[Z_i | \theta] \quad .$$

Then

$$E[N^2] = \frac{E[Z^2 | \theta] - E[N]\ V[Z_i | \theta]}{E^2[Z_i | \theta]} \quad , \tag{5}$$

with

$$E[Z^2 | \theta] \doteq [1-L(\theta)]Ln^2A + L(\theta)Ln^2B \quad .$$

Using Eqs. (3) and (5) in (4), an approximate Variance of the Sample Size is thus given by

$$V[N] = \frac{E[Z^2 | \theta] - E[N]V[Z_i | \theta] - E^2[Z | \theta]}{E^2[Z_i | \theta]}$$

$$V[N] = \frac{V[Z | \theta] - E[N]\ V[Z_i | \theta]}{E^2[Z_i | \theta]} \quad . \tag{6}$$

Unfortunately, using Eq. (6) to calculate the approximate Variance of Sample Size leads to negative values in many cases. This appears to be caused by the magnification of the approximations used in Eq. (3) when they are entered in Eq. (5). Whether this is in fact the true cause, the approximation in general is not good. Consequently, no numerical tabulations of the V[N]using Eq. (6) are included. However, emperically determined "exact" values of the V[N] are included.

IV. DISTRIBUTIONS INVESTIGATED

The author investigated the SPRT for three common distributions: the Binomial, Normal, and Exponential. Each distribution was investigated for various parameter values. Each specific distribution was used in calculating the Wald "approximate" results and in generating empirical "exact" results for the OC curve, Expected Sample Size, and the Variance of Sample Size.

Explicit equations for the points on the Wald "approximate" Operating Characteristic Curve, the "approximate" Expected Sample Size Curve, and the "approximate" Variance of Sample Size Curve are given for these distributions using Eqs. (1), (3), and (6).

A. BINOMIAL DISTRIBUTION

Suppose $f(x;\ \theta) = \theta^{x}\ (1-\theta)^{1-x}$ the logarithm of the likelihood ratio is

$$Z_i = \mathrm{Ln}\left[\left(\frac{\theta_1}{\theta_o}\right)\left(\frac{1-\theta_1}{1-\theta_o}\right)^{1-X_i}\right] \quad .$$

Then

$$E[Z_i \mid \theta] = \theta \mathrm{Ln}\ \frac{f(1,\ \theta_1)}{f(1,\ \theta_o)} + (1-\theta)\ \mathrm{Ln}\ \frac{f(0,\ \theta_1)}{f(0,\ \theta_o)}$$

$$E[Z_i \mid \theta] = \theta \mathrm{Ln}\ \left[\frac{\theta_1}{\theta_o}\right] + (1-\theta)\ \mathrm{Ln}\left[\frac{1-\theta_1}{1-\theta_o}\right] \tag{7}$$

and

$$V[Z_i \mid \theta] = E[Z_i^2 \mid \theta] + E^2[Z_i \mid \theta]$$

$$V[Z_i \mid \theta] = \theta(1-\theta) \left[\operatorname{Ln} \frac{\theta_1}{\theta_o} \frac{(1-\theta_o)}{(1-\theta_1)} \right]^2 . \tag{8}$$

In order to obtain an approximate OC function, the expression of Eq. (2) for this case,

$$\theta \left[\frac{\theta_1}{\theta_o}\right]^{h(\theta)} + (1-\theta) \left[\frac{(1-\theta_1)}{(1-\theta_o)}\right]^{h(\theta)} = 1 ,$$

was solved for θ and evaluated with various selected values of $h(\theta)$,

$$\theta = \frac{1 - \left[\frac{(1-\theta_1)}{(1-\theta_o)}\right]^{h(\theta)}}{\left[\frac{\theta_1}{\theta_o}\right]^{h(\theta)} - \left[\frac{(1-\theta_1)}{(1-\theta_o)}\right]^{h(\theta)}} . \tag{9}$$

Utilizing Eq. (9) with Eq. (1) points on the Wald approximate OC Curve were obtained.

By substituting Eq. (7) into Eq. (3) the Wald approximate Sample Size curve was determined. An attempt was made to determine the approximate Variance of Sample Size, utilizing Eqs. (7) and (8) with the prior results of E[N], in Eq. (6).

The acceptance and rejection numbers were respectively computed from

$$A_n = \frac{\text{Ln } B + n\text{Ln } \frac{(1-\theta_o)}{(1-\theta_1)}}{\text{Ln } \frac{\theta_1}{\theta_o} - \text{Ln } \frac{(1-\theta_1)}{(1-\theta_o)}}$$

and

$$Rn = \frac{\text{Ln } A + n\text{Ln } \frac{(1-\theta_o)}{(1-\theta_1)}}{\text{Ln } \frac{\theta_1}{\theta_o} - \text{Ln } \frac{(1-\theta_1)}{(1-\theta_o)}} \quad .$$

B. NORMAL DISTRIBUTION

If $f(x;\ \theta)$ is Normal $(\theta,\ \sigma^2)$ then the log likelihood ratio is

$$Z_i = \frac{1}{2\sigma^2} \ [2(\theta_1-\theta_o)X_i + (\theta_o^{\ 2}-\theta_1^{\ 2})],$$

and

$$E[Z_i |\ \theta] = \frac{\theta_o^{\ 2}-\theta_1^{\ 2}}{2\sigma^2} + \frac{\theta(\theta_1-\theta_o)}{\sigma^2} \tag{10}$$

and

$$V[Z_i |\ \theta] = \frac{(\theta_1-\theta_o)^2}{\sigma^2} \quad . \tag{11}$$

Substituting Eq. (10) into Eq. (3) points on the Wald approximate Sample Size Curve were computed. Eqs. (10) and (11) with $E[N]$ computed above were used in Eq. (6) in an attempt to determine the approximate Variance of N.

In order to determine points on the approximate OC curve, the expression of Eq. (2) for the present case was solved for $h(\theta)$, resulting in

$$h(\theta) = \frac{\theta_1 + \theta_o - 2\theta}{\theta_1 - \theta_o} \quad .$$

The acceptance and rejection numbers are, in this case, respectively given by

$$A_n = [\sigma^2/(\theta_1-\theta_o)]\text{Ln } B + n[\frac{\theta_o+\theta_1}{2}]$$

and

$$R_n = [\sigma^2/(\theta_1-\theta_o)]\text{Ln } A + n[\frac{\theta_o+\theta_1}{2}] \quad .$$

C. EXPONENTIAL DISTRIBUTION

If $f(x;\ \theta) = \theta\ e^{-\theta x}$; $x \geq 0$, the log likelihood ratio is

$$Z_i = \text{Ln } \frac{\theta_1}{\theta_o} - (\theta_1-\theta_o)X_i$$

so that

$$E[Z_i|\ \theta] = \text{Ln } \frac{\theta_1}{\theta_o} - \frac{(\theta_1-\theta_o)}{\theta} \tag{12}$$

and

$$V[Z_i|\ \theta] = \frac{(\theta_1-\theta_o)^2}{\theta^2} \tag{13}$$

Substituting Eq. (12) into Eq. (3), points on the Wald approximate Sample Size curve were obtained. Eqs. (12) and (13) were used in Eq. (6) in an attempt to determine an approximate Variance of N.

In order to determine values for an approximate OC curve, Eq. (2) was solved for θ, resulting in

$$\theta = \frac{h(\theta)\,(\theta_1-\theta_o)}{\left[\frac{\theta_1}{\theta_o}\right]^{h(\theta)} - 1} \quad . \tag{14}$$

Utilizing Eqs. (1) and (14) points on the Wald approximate OC Curve were obtained.

The acceptance and rejection numbers are respectively given by

$$A_n = \frac{-\mathrm{Ln}\ B + n\mathrm{Ln}\ \frac{\theta_1}{\theta_o}}{(\theta_1-\theta_o)}$$

and

$$R_n = \frac{\mathrm{Ln}\ A + n\mathrm{Ln}\ \frac{\theta_1}{\theta_o}}{(\theta_1-\theta_o)} \quad .$$

V. PROCEDURE

Computer programs, coded in FORTRAN IV, were written which gives points on the Wald approximate OC Curves, and the approximate Expected Sample Size curves. Programs for the Binomial, Normal, and Exponential distributions, each entitled "Approximate (Distribution)" are listed under Computer Programs.

In order to evaluate the Wald approximations, companion programs called "exact" programs were written. These programs produce Monte Carlo simulation of the SPRT procedure for the distribution considered. Each program determines "exact" points on the OC curve, Expected Sample Size curve and Variance of Sample Size curve.

The simulation models sequential sampling from a specific known distribution with parameter values being inputs to the simulation. As each sample was observed, a corresponding acceptance and rejection number was computed and the test statistic was computed. These values were compared and the appropriate alternative was selected.

A. DETERMINATION OF THE NUMBER OF REPLICATIONS

In order for the simulation estimates of the Operating Characteristic points, $\hat{L}(\theta)$, (a Bernoulli parameter) to be useful, it is necessary to use a large number of replications (that is, simulate the performance of many tests) so that with a high probability $\hat{L}(\theta)$ is "close" to the true

$L(\theta)$. A Normal approximation to the binomial was used to determine the number of replications that should be used at each sample point. This was done as follows:

Let S_J denote the number of successes in J independent repeated Bernoulli trials with parameter p, and let

$$P_J = \frac{1}{J} S_J$$

denote the average number of successes in J trials. For large J, P_J can be shown to be approximately Normal with mean p and variance $p(1-p)/J$. Then $Y_J = (P_J - p)/[p(1-p)/J]^{\frac{1}{2}}$ is approximately Normal (0, 1). For any level of risk, $\gamma > 0$, and minimum acceptable probability bound, $\delta > 0$, we seek J such that

$$P[P_J - p] = P\left[\left|\frac{P_{J-p}}{\sqrt{p(1-p)/J}}\right| \leq \frac{\gamma}{\sqrt{p(1-p)/J}}\right] \geq \delta$$

or

$$P\left[\frac{-\gamma\sqrt{J}}{\sqrt{p(1-p)}} \leq Y_J \leq \frac{\gamma\sqrt{J}}{\sqrt{p(1-p)}}\right] \geq \delta \quad .$$

This occurs whenever

$$J \geq \frac{y_\delta^{\,2}\ p\ (1-p)}{\gamma^2}$$

where y_δ is such that $P[Y < -y_\delta] = \frac{1-\delta}{2}$, where Y is distributed Normal (0, 1). At $p = 1-p = 1/2$, $J \geq \frac{y_\delta^{\,2}}{(2\gamma)^2}$, so for $\gamma = .01$ and $\delta = .95$, $Y_\delta = 1.645$ and the required value of J is approximately 6,765. Additionally:

$p = .1, \quad \gamma = .01, \quad \delta = .95$ requires $J = 1481$;

$p = .01, \quad \gamma = .01, \quad \delta = .95$ requires $J = 163$;

$p = .1, \quad \gamma = .01, \quad \delta = .95$ requires $J = 2095$;

$p = .3, \quad \gamma = .01, \quad \delta = .95$ requires $J = 4889$.

In view of these and similar determinations and since it was desired to estimate $L(\theta)$ values as large as .30 to within reasonable accuracy, a selection of 5000 replications for each estimation of $L(\theta)$ value was made. The values obtained in the simulation should therefore, with high probability, be accurate to at least 2, and usually 3, decimal places. A fourth decimal was carried in the tables to exclude possible round off errors.

B. COMPUTER SIMULATION

For each distribution investigated two computer programs were developed; an "approximate program" based on the Wald approximations for the SPRT, and the "exact program" involving a Monte Carlo simulation. The inputs necessary for these programs are the number of distributions to be inspected, the parameter values for the Null and Alternate Hypotheses (θ_o and θ_1), the "target" Type I and Type II Error Probabilities (α and β), parameters for the distribution being sampled (θ), arguments for the random number generator (URN, a special random number generator included in the IBM Scientific Package at the Naval Postgraduate School), the number of replications (in this thesis 5000) at each test point, (θ), and the number of test points from the "approximate program" being used in the "exact program." These values, punched in one input card, and the associated output from the "approximate program," which is run first, comprise the input data for the exact program.

Simulation of observations from Binomial, Normal, and Exponential distributions were by standard methods. Background information may be obtained in [Naylor, 1967] and [McMillan and Gonzalez, 1968].

In order to obtain the values tabulated in this thesis the procedure was to first run an "approximate program." These programs produce a fixed number of test points. In the Approximate Binomial program the number of test points (now 85) is controlled by the last "IF" statement in the program, varying the value 41 changes the number of test points. Control of the number of test points (80) in the Approximate Normal program is by changing the numerical multipliers in the first "XM2" and "XM" steps. Control of the number of test points (50) in the Approximate Exponential program is by changing the limit on the "DO 7" statement. The presentation of the Approximate Program output (Approximate OC Value, θ, Approximate Expected Sample Size) is a listing and a separate punched card for each test point, θ. Prior to running the "exact program," the number of test points was reduced to 15 to 20 so as to reduce the execution time. The Exact Program output (see tables) is a listing and a set of punched cards.

"Exact" OC values of various SPRT's were estimated by the relative frequency of the number of replications terminating with acceptance of H_o, at each of several values of θ.

"Exact" expected sample sizes at each test point, θ, were estimated by tallying, each time the test was performed, the observed sample requirement. The average sample size being computed at the end of the 5000 replications. These averages were taken to be the exact values of E[N].

The "exact" variance of sample size at each test point, θ, was computed in a manner similar to that used for the E[N],

$$V[N] = \frac{1}{n-1} \left[\sum_{i=1}^{n} m_i^2 - nE^2[N] \right] ,$$

where n = 5000 replications and m_i denotes the number of observations required in the ith simulated test.

C. COMPUTER STATISTICS.

The IBM-360 at the Naval Postgraduate School was utilized in computing the values presented. The maximum core space needed for any one of the programs was less than 58K bytes. The necessary time to complete an "approximate" case was less than 5 seconds. The "exact" programs were run under the H compiler of the IBM-360. In certain cases this allowed an execution time savings of 80% with respect to running under the G compiler. The running time per case for the programs averaged 11 minutes as presented, with a maximum of about 15 minutes.

VI. CONCLUSIONS AND RESULTS

A comparison of the Wald "approximate" results with the "exact" results described above confirmed the known fact that, in all cases, the results obtained using the Wald approximations for the SPRT are conservative in that a given test plan's error probabilities are greater than the exact values. In the Binomial cases as much as a 22% difference in the target α level and the exact α level was noted; for the Exponential distribution in one case ($H_o = 1.0$, $H_1 = 5.0$, $\alpha = .03$, $\beta = .05$) a 50% difference was noted, with the differences generally averaging about 25 to 30%. For the Normal distribution the percentage difference in the target α level and the exact α level appears to increase with $|\theta_o - \theta_1|$.

A comparison of the approximate and exact expected sample sizes is facilitated by the tabilized values. In all cases the results following indicate that at each test point the "exact" expected sample size is larger than the Wald approximation.

The exact variances of sample size found here support the conjecture of [Cox and Roseberry, 1966a], namely, that V[N] is approximately the square of E[N]. Unfortunately, as noted earlier a natural approximation for V[N] turned out to yield especially poor approximations. However, the exact variance of N does appear to increase roughly as the square of E[N], in the cases considered here.

In all, forty-six cases were examined. The final results of all cases are tabulated below.

BINOMIAL SEQUENTIAL SAMPLING

TEST: NULL HYPOTHESIS: THETA = 0.150

TEST: ALTERNATE HYPOTHESIS: THETA = 0.300

TYPE I ERROR = 0.010 PROB(REJECT NULL HYP. GIVEN NULL HYP. TRUE)

TYPE II ERROR = 0.010 PROB(ACCEPT NULL HYP. GIVEN ALT. HYP. TRUE)

THETA	OPERATING CHARACTERISTIC VALUE			EXPECTED SAMPLE SIZE			VARIANCE S S
	APPROX.	EXACT	DIFFERENCE	APPROX.	EXACT	DIFFERENCE	EXACT
0.0	1.0000	1.0000	0.0	23.6671	24.0000	0.3329	0.0
0.1324	0.9975	0.9980	0.0005	59.5946	60.6602	1.0656	903.19
0.1500	0.9900	0.9906	0.0006	73.7495	74.5914	0.8419	1802.71
0.1826	0.9087	0.9218	0.0131	116.7943	121.8060	5.0117	7512.95
0.1967	0.7988	0.8128	0.0140	139.6510	147.7166	8.0656	12555.26
0.2113	0.6129	0.6256	0.0127	155.5712	167.9512	12.3800	17680.81
0.2183	0.5000	0.5064	0.0064	156.8974	170.8722	13.9748	19632.52
0.2265	0.3871	0.3890	0.0019	152.8729	165.0696	12.1967	18097.79
0.2421	0.2012	0.1926	-.0086	132.8153	145.0544	12.2391	13703.27
0.2582	0.0913	0.0766	-.0147	107.4805	116.4164	8.9359	8423.50
0.3000	0.0100	0.0060	-.0040	62.5146	66.3754	3.8608	1874.84
0.3086	0.0063	0.0050	-.0013	56.9609	60.0380	3.0771	1431.56
1.0000	0.0	0.0	0.0	6.6294	7.0000	0.3706	0.0

BINOMIAL SEQUENTIAL SAMPLING

TEST: NULL HYPOTHESIS: THETA = 0.150

TEST: ALTERNATE HYPOTHESIS: THETA = 0.300

TYPE I ERROR = 0.010 PROB(REJECT NULL HYP. GIVEN NULL HYP. TRUE)

TYPE II ERROR = 0.100 PROB(ACCEPT NULL HYP. GIVEN ALT. HYP. TRUE)

THETA	OPERATING CHARACTERISTIC VALUE			EXPECTED SAMPLE SIZE			VARIANCE S S
	APPROX	EXACT	DIFFERENCE	APPROX.	EXACT	DIFFERENCE	EXACT
0.0	1.0000	1.0000	0.0	11.8077	12.0000	0.1923	0.0
0.1324	0.9973	0.9984	0.0011	29.6417	30.9970	1.3553	470.27
0.1500	0.9900	0.9910	0.0010	36.4326	37.9810	1.5484	864.60
0.1826	0.9256	0.9366	0.0110	55.5757	59.0324	3.4567	2883.38
0.1967	0.8518	0.8628	0.0110	65.3884	70.2434	4.8550	3940.39
0.2113	0.7350	0.7466	0.0116	73.8716	81.8406	7.9690	5509.69
0.2188	0.6625	0.6696	0.0071	76.6536	85.3308	8.6772	5754.69
0.2265	0.5844	0.5860	0.0016	78.1113	87.0608	8.9495	5985.10
0.2421	0.4282	0.4212	-.0070	76.9855	85.0554	8.0699	5306.84
0.2582	0.2942	0.2772	-.0170	71.5906	79.1542	7.5636	3883.44
0.2747	0.1940	0.1792	-.0148	64.2044	70.0062	5.8018	2528.15
0.3000	0.1000	0.0930	-.0070	53.0381	56.8070	3.7689	1327.11
0.3259	0.0506	0.0394	-.0112	43.7379	47.0242	3.2863	726.57
1.0000	0.0	0.0	0.0	6.4918	7.0000	0.5082	0.0

BINOMIAL SEQUENTIAL SAMPLING

TEST: NULL HYPOTHESIS: THETA = 0.150

TEST: ALTERNATE HYPOTHESIS: THETA = 0.300

TYPE I ERROR = 0.010 PROB(REJECT NULL HYP. GIVEN NULL HYP. TRUE)

TYPE II ERROR = 0.200 PROB(ACCEPT NULL HYP. GIVEN ALT. HYP. TRUE)

THETA	OPERATING CHARACTERISTIC VALUE			EXPECTED SAMPLE SIZE			VARIANCE S S
	APPROX.	EXACT	DIFFERENCE	APPROX.	EXACT	DIFFERENCE	EXACT
0.0	1.0000	1.0000	0.0	8.2376	9.0000	0.7624	0.0
0.1324	0.9971	0.9980	0.0009	20.6193	22.1414	1.5221	324.31
0.1500	0.9900	0.9910	0.0010	25.2137	26.9784	1.7647	573.50
0.1967	0.8772	0.8824	0.0052	44.0012	48.9316	4.9304	2458.36
0.2113	0.7882	0.7954	0.0072	49.8244	56.4642	6.6398	3200.17
0.2188	0.7326	0.7368	0.0042	52.0776	59.0192	6.9416	3525.31
0.2265	0.6717	0.6710	-.0007	53.7046	60.8792	7.1746	3526.26
0.2421	0.5429	0.5292	-.0137	54.8842	62.4194	7.5352	3210.07
0.2582	0.4203	0.4054	-.0149	53.4514	59.2962	5.8448	2488.42
0.2747	0.3160	0.2956	-.0204	50.2743	55.7886	5.5143	1807.12
0.3000	0.2000	0.1822	-.0178	44.2252	48.2904	4.0652	1070.69
0.3259	0.1247	0.1088	-.0159	38.2706	41.4352	3.1646	640.60
1.0000	0.0	0.0	0.0	6.3219	7.0000	0.6781	0.0

BINOMIAL SEQUENTIAL SAMPLING

TEST: NULL HYPOTHESIS: THETA = 0.150

TEST: ALTERNATE HYPOTHESIS: THETA = 0.300

TYPE I ERROR = 0.100 PROB(REJECT NULL HYP. GIVEN NULL HYP. TRUE)

TYPE II ERROR = 0.010 PROB(ACCEPT NULL HYP. GIVEN ALT. HYP. TRUE)

THETA	OPERATING CHARACTERISTIC VALUE			EXPECTED SAMPLE SIZE			VARIANCE S S
	APPROX.	EXACT	DIFFERENCE	APPROX.	EXACT	DIFFERENCE	EXACT
0.0	1.0000	1.0000	0.0	23.1762	24.0000	0.8238	0.0
0.1324	0.9494	0.9620	0.0126	54.1727	56.3304	2.1577	767.39
0.1500	0.9000	0.9228	0.0228	62.5698	65.9816	3.4118	1315.62
0.1691	0.8060	0.8390	0.0330	72.1093	76.9894	4.8801	2339.41
0.1826	0.7058	0.7452	0.0394	77.7943	85.7878	7.9935	3675.15
0.1967	0.5718	0.6062	0.0344	80.9479	90.1744	9.2265	4978.64
0.2113	0.4156	0.4436	0.0280	79.4901	90.6990	11.2089	6073.67
0.2188	0.3375	0.3588	0.0213	76.6536	85.5902	8.9366	5782.96
0.2265	0.2650	0.2790	0.0140	72.5904	83.7806	11.1902	6263.51
0.2421	0.1482	0.1492	0.0010	62.1877	71.6994	9.5117	4998.82
0.2582	0.0744	0.0740	-.0004	51.1439	57.6096	6.4657	3093.89
0.3000	0.0100	0.0072	-.0028	30.8825	34.1062	3.2237	962.39
0.3086	0.0065	0.0052	-.0013	28.2271	30.9362	2.7091	685.41
1.0000	0.0	0.0	0.0	3.3074	4.0000	0.6926	0.0

BINOMIAL SEQUENTIAL SAMPLING

TEST: NULL HYPOTHESIS: THETA = 0.150

TEST: ALTERNATE HYPOTHESIS: THETA = 0.300

TYPE I ERROR = 0.100 PROB(REJECT NULL HYP. GIVEN NULL HYP. TRUE)

TYPE II ERROR = 0.100 PROB(ACCEPT NULL HYP. GIVEN ALT. HYP. TRUE)

THETA	OPERATING CHARACTERISTIC VALUE APPROX	EXACT	DIFFERENCE	EXPECTED SAMPLE SIZE APPROX.	EXACT	DIFFERENCE	VARIANCE S S EXACT
0.0	1.0000	1.0000	0.0	11.3168	12.0000	0.6832	0.0
0.1324	0.9456	0.9598	0.0142	25.5280	27.7718	2.2438	201.64
0.1500	0.9000	0.9216	0.0216	28.7873	31.5674	2.7801	463.95
0.1631	0.8232	0.8598	0.0366	32.1828	36.9580	4.7752	735.03
0.1826	0.7500	0.7760	0.0260	34.1636	39.2646	5.1010	862.94
0.1967	0.6591	0.6802	0.0211	35.5555	40.8904	5.3349	1004.25
0.2188	0.5000	0.5160	0.0160	35.8733	42.0052	6.1319	1180.89
0.2421	0.3409	0.3400	-.0009	33.8145	39.3342	5.5197	1056.47
0.2582	0.2500	0.2422	-.0078	31.4389	36.8220	5.3831	875.65
0.2747	0.1768	0.1644	-.0124	28.6547	33.3836	4.7289	700.22
0.3000	0.1000	0.0852	-.0143	24.4019	28.0030	3.6011	456.30
0.3259	0.0544	0.0438	-.0106	20.6107	23.2650	2.6543	290.30
1.0000	0.0	0.0	0.0	3.1699	4.0000	0.8301	0.0

BINOMIAL SEQUENTIAL SAMPLING

TEST: NULL HYPOTHESIS: THETA = 0.150

TEST: ALTERNATE HYPOTHESIS: THETA = 0.300

TYPE I ERROR = 0.100 PROB(REJECT NULL HYP. GIVEN NULL HYP. TRUE)

TYPE II ERROR = 0.200 PROB(ACCEPT NULL HYP. GIVEN ALT. HYP. TRUE)

THETA	OPERATING CHARACTERISTIC VALUE			EXPECTED SAMPLE SIZE			VARIANCE S S
	APPROX.	EXACT	DIFFERENCE	APPROX.	EXACT	DIFFERENCE	EXACT
0.0	1.0000	1.0000	0.0	7.7467	8.0000	0.2533	0.0
0.1324	0.9419	0.9564	0.0145	16.8941	18.6858	1.7917	182.05
0.1500	0.9000	0.9208	0.0208	18.7636	21.2048	2.4412	258.26
0.1691	0.8347	0.8612	0.0265	20.6585	24.2090	3.5505	380.60
0.1826	0.7757	0.7920	0.0163	21.7811	25.3680	3.5869	440.81
0.1967	0.7046	0.7186	0.0140	22.6539	26.8948	4.2409	491.71
0.2188	0.5803	0.5834	0.0031	23.2402	27.3580	4.1178	507.88
0.2421	0.4487	0.4416	-.0071	22.8081	27.0402	4.2321	513.21
0.2582	0.3657	0.3558	-.0099	22.0064	25.7706	3.7642	446.31
0.2747	0.2913	0.2704	-.0203	20.8973	24.8138	3.9165	389.66
0.3000	0.2000	0.1766	-.0234	18.9178	22.1350	3.2172	269.29
0.3434	0.1006	0.0882	-.0124	15.5510	17.6564	2.1054	155.70
1.0000	0.0	0.0	0.0	3.0000	3.0000	0.0	0.0

BINOMIAL SEQUENTIAL SAMPLING

TEST: NULL HYPOTHESIS: THETA = 0.100

TEST: ALTERNATE HYPOTHESIS: THETA = 0.200

TYPE I ERROR = 0.100 PROB(REJECT NULL HYP. GIVEN NULL HYP. TRUE)

TYPE II ERROR = 0.100 PROB(ACCEPT NULL HYP. GIVEN ALT. HYP. TRUE)

THETA	OPERATING CHARACTERISTIC VALUE			EXPECTED SAMPLE SIZE			VARIANCE S S
	APPROX.	EXACT	DIFFERENCE	APPROX.	EXACT	DIFFERENCE	EXACT
0.0	1.0000	1.0000	0.0	18.6548	19.0000	0.3452	0.0
0.0885	0.9456	0.9624	0.0168	42.5574	45.4348	2.8774	782.71
0.1000	0.9000	0.9220	0.0220	47.9083	51.6844	3.7761	1189.71
0.1125	0.8232	0.8552	0.0320	53.4442	59.1180	5.6738	1796.04
0.1213	0.7500	0.7824	0.0324	56.6330	62.8294	6.1964	2260.67
0.1306	0.6591	0.6926	0.0335	58.8252	66.9458	8.1206	2791.27
0.1452	0.5000	0.5248	0.0248	59.1342	68.0952	8.9610	3120.19
0.1608	0.3409	0.3560	0.0151	55.5030	64.7794	9.2764	2979.59
0.1716	0.2500	0.2546	0.0046	51.4498	60.5158	9.0660	2416.68
0.1827	0.1768	0.1768	0.0	46.7377	54.3044	7.5667	1925.95
0.2000	0.1000	0.0894	-.0106	39.5873	45.7968	6.2095	1287.03
0.2179	0.0544	0.0488	-.0056	33.2353	37.7494	4.5141	782.12
1.0000	0.0	0.0	0.0	3.1699	4.0000	0.8301	0.0

BINOMIAL SEQUENTIAL SAMPLING

TEST: NULL HYPOTHESIS: THETA = 0.300

TEST: ALTERNATE HYPOTHESIS: THETA = 0.700

TYPE I ERROR = 0.100 PROB(REJECT NULL HYP. GIVEN NULL HYP. TRUE)

TYPE II ERROR = 0.100 PROB(ACCEPT NULL HYP. GIVEN ALT. HYP. TRUE)

THETA	OPERATING CHARACTERISTIC VALUE			EXPECTED SAMPLE SIZE			VARIANCE S S
	APPROX	EXACT	DIFFERENCE	APPROX.	EXACT	DIFFERENCE	EXACT
0.0	1.0000	1.0000	0.0	2.5932	3.0000	0.4068	0.0
0.2495	0.9456	0.9606	0.0150	4.6127	5.6128	1.0001	12.27
0.3000	0.9000	0.9190	0.0190	5.1864	6.4428	1.2564	18.02
0.3559	0.8232	0.8438	0.0206	5.8168	7.4040	1.5872	26.04
0.3956	0.7500	0.7646	0.0146	6.2123	8.0864	1.8741	36.68
0.4368	0.6591	0.6686	0.0095	6.5261	8.5832	2.0571	42.63
0.5000	0.5000	0.4852	-.0148	6.7248	8.9568	2.2320	49.13
0.5632	0.3409	0.3096	-.0313	6.5265	8.5892	2.0627	42.17
0.6044	0.2500	0.2112	-.0388	6.2125	8.0768	1.8643	37.10
0.6441	0.1768	0.1402	-.0366	5.8169	7.3928	1.5759	28.43
0.7000	0.1000	0.0714	-.0286	5.1864	6.3668	1.1804	17.84
0.7505	0.0544	0.0340	-.0204	4.6127	5.4780	0.8653	10.68
1.0000	0.0	0.0	0.0	2.5932	3.0000	0.4068	0.0

EXPONENTIAL SEQUENTIAL SAMPLING

F(X, T) = T * EXP(-T*X)

TEST: NULL HYPOTHESIS: THETA = 1.000

TEST: ALTERNATE HYPOTHESIS: THETA = 1.500

TYPE I ERROR = 0.010 PROB(REJECT NULL HYP. GIVEN NULL HYP. TRUE)

TYPE II ERROR = 0.010 PROB(ACCEPT NULL HYP. GIVEN ALT. HYP. TRUE)

THETA	OPERATING CHARACTERISTIC VALUE			EXPECTED SAMPLE SIZE			VARIANCE S S
	APPROX	EXACT	DIFFERENCE	APPROX.	EXACT	DIFFERENCE	EXACT
0.8000	0.9999	1.0000	0.0001	20.9269	23.4684	2.5415	174.72
0.8959	0.9990	0.9996	0.0006	30.0485	33.3068	3.2583	426.15
1.0000	0.9900	0.9922	0.0022	47.6355	52.8936	5.2581	1328.97
1.0439	0.9753	0.9804	0.0051	59.4410	66.2848	6.8438	2240.46
1.0892	0.9403	0.9438	0.0035	75.5304	84.3584	8.8280	4131.66
1.1359	0.8627	0.8712	0.0085	95.9765	108.0084	12.0319	7356.29
1.1838	0.7148	0.7218	0.0070	116.8752	134.9108	18.0356	11854.34
1.2083	0.6129	0.6068	-.0061	124.4879	141.4762	16.9883	13603.69
1.2583	0.3871	0.3662	-.0209	127.9383	144.9472	17.0089	13659.70
1.2833	0.2852	0.2612	-.0240	123.3672	138.2190	14.8518	11958.42
1.3359	0.1373	0.1158	-.0215	106.9357	116.8130	9.8773	7343.23
1.3892	0.0597	0.0488	-.0109	88.8327	93.7094	4.8767	3927.51
1.4439	0.0247	0.0176	-.0071	73.7970	77.1718	3.3748	2196.74
1.5000	0.0100	0.0066	-.0034	62.4304	64.9020	2.4716	1278.35
1.6459	0.0010	0.0010	0.0	45.0978	46.6076	1.5098	395.19
1.8000	0.0001	0.0	-.0001	35.9799	37.2338	1.2539	168.81

EXPONENTIAL SEQUENTIAL SAMPLING

F(X, T) = T * EXP(-T*X)

TEST: NULL HYPOTHESIS: THETA = 1.000

TEST: ALTERNATE HYPOTHESIS: THETA = 1.500

TYPE I ERROR = 0.010 PROB(REJECT NULL HYP. GIVEN NULL HYP. TRUE)

TYPE II ERROR = 0.100 PROB(ACCEPT NULL HYP. GIVEN ALT. HYP. TRUE)

THETA	OPERATING CHARACTERISTIC VALUE			EXPECTED SAMPLE SIZE			VARIANCE S S
	APPROX.	EXACT	DIFFERENCE	APPROX.	EXACT	DIFFERENCE	EXACT
0.8000	0.9999	1.0000	0.0001	10.4389	12.9706	2.5317	91.25
0.8959	0.9989	0.9994	0.0005	14.9714	18.2698	3.2984	222.25
1.0000	0.9900	0.9928	0.0028	23.5322	28.3302	4.7980	663.47
1.0439	0.9769	0.9782	0.0013	29.0647	34.9988	5.9341	1046.09
1.0892	0.9489	0.9508	0.0019	36.3129	44.1140	7.8011	1760.27
1.1359	0.8937	0.8944	0.0007	45.2265	55.7500	10.5235	2886.55
1.1838	0.7987	0.7888	-.0099	54.7809	68.3978	13.6169	4243.99
1.2083	0.7350	0.7140	-.0210	59.1122	73.3980	14.2858	4788.88
1.2583	0.5844	0.5446	-.0398	65.3707	79.4650	14.0943	4589.87
1.2838	0.5050	0.4530	-.0520	66.8434	81.5030	14.6596	4524.34
1.3359	0.3572	0.3032	-.0540	66.5109	77.8754	11.3645	3261.21
1.3892	0.2398	0.1882	-.0516	63.0267	71.5446	8.5179	2201.41
1.4439	0.1561	0.1100	-.0461	58.1102	65.0646	6.9544	1454.71
1.5000	0.1000	0.0668	-.0332	52.9666	57.6856	4.7190	982.68
1.6459	0.0321	0.0168	-.0153	42.1102	44.5868	2.4766	369.98
1.8000	0.0102	0.0044	-.0058	34.6982	36.2714	1.5732	167.21

EXPONENTIAL SEQUENTIAL SAMPLING

F(X, T) = T * EXP(-T*X)

TEST: NULL HYPOTHESIS: THETA = 1.000

TEST: ALTERNATE HYPOTHESIS: THETA = 1.500

TYPE I ERROR = 0.100 PROB(REJECT NULL HYP. GIVEN NULL HYP. TRUE)

TYPE II ERROR = 0.010 PROB(ACCEPT NULL HYP. GIVEN ALT. HYP. TRUE)

THETA	OPERATING CHARACTERISTIC VALUE			EXPECTED SAMPLE SIZE			VARIANCE S S
	APPROX.	EXACT	DIFFERENCE	APPROX.	EXACT	DIFFERENCE	EXACT
0.8000	0.9898	0.9924	0.0026	20.1814	22.8188	2.6374	164.43
0.8959	0.9679	0.9728	0.0049	28.0579	31.5264	3.4685	358.44
1.0000	0.9000	0.9182	0.0182	40.4144	46.1442	5.7298	938.33
1.0439	0.8439	0.8602	0.0163	46.8058	53.3902	6.5844	1425.10
1.0892	0.7602	0.7790	0.0188	53.5887	61.9562	8.3675	2223.07
1.1359	0.6428	0.6594	0.0166	59.6946	69.8480	10.1534	3203.59
1.1838	0.4950	0.5002	0.0052	63.3260	74.8640	11.5380	3998.87
1.2083	0.4156	0.4160	0.0004	63.6079	75.5810	11.9731	4554.62
1.2583	0.2650	0.2520	-.0130	60.7506	71.0604	10.3098	4102.04
1.2838	0.2013	0.1866	-.0147	57.8238	67.0824	9.2586	3659.24
1.3359	0.1063	0.0942	-.0121	50.3907	56.2456	5.8549	2534.03
1.3892	0.0511	0.0408	-.0103	42.7082	47.2066	4.4984	1585.19
1.4439	0.0231	0.0168	-.0063	36.0843	39.5664	3.4821	1013.25
1.5000	0.0100	0.0062	-.0038	30.8409	33.4382	2.5973	621.09
1.6459	0.0011	0.0006	-.0005	22.4696	24.2166	1.7470	210.36
1.8000	0.0001	0.0002	0.0001	17.9478	19.2350	1.2872	90.25

EXPONENTIAL SEQUENTIAL SAMPLING

F(X, T) = T * EXP(-T*X)

TEST: NULL HYPOTHESIS: THETA = 1.000

TEST: ALTERNATE HYPOTHESIS: THETA = 1.500

TYPE I ERROR = 0.100 PROB(REJECT NULL HYP. GIVEN NULL HYP. TRUE)

TYPE II ERROR = 0.100 PROB(ACCEPT NULL HYP. GIVEN ALT. HYP. TRUE)

THETA	OPERATING CHARACTERISTIC VALUE			EXPECTED SAMPLE SIZE			VARIANCE S S
	APPROX.	EXACT	DIFFERENCE	APPROX.	EXACT	DIFFERENCE	EXACT
0.8000	0.9878	0.9930	0.0052	9.7644	12.4180	2.6536	83.08
0.8959	0.9643	0.9730	0.0087	13.3690	16.7980	3.4290	168.05
1.0000	0.9000	0.9162	0.0162	18.5940	23.3862	4.7922	367.01
1.0439	0.8529	0.8636	0.0107	21.1049	26.4402	5.3353	457.95
1.0892	0.7889	0.7988	0.0099	23.6970	30.1388	6.4418	601.24
1.1359	0.7066	0.7082	0.0016	26.1389	33.1298	6.9909	720.90
1.1838	0.6081	0.6000	-.0081	28.1262	36.3848	8.2586	938.44
1.2083	0.5547	0.5396	-.0151	28.8460	36.4566	7.6106	861.72
1.2583	0.4453	0.4174	-.0279	29.6455	37.2774	7.6319	863.46
1.2838	0.3919	0.3576	-.0343	29.6886	37.3928	7.7042	880.25
1.3359	0.2934	0.2564	-.0370	29.1236	35.5548	6.4312	750.53
1.3892	0.2111	0.1686	-.0425	27.8705	33.7246	5.8541	619.57
1.4439	0.1471	0.1092	-.0379	26.2020	30.9532	4.7512	488.83
1.5000	0.1000	0.0698	-.0302	24.3690	28.1348	3.7658	347.86
1.6459	0.0357	0.0230	-.0127	20.0645	22.1652	2.1007	159.72
1.8000	0.0122	0.0054	-.0068	16.7881	18.3058	1.5177	79.97

EXPONENTIAL SEQUENTIAL SAMPLING

F(X, T) = T * EXP(-T*X)

TEST: NULL HYPOTHESIS: THETA = 1.000

TEST: ALTERNATE HYPOTHESIS: THETA = 2.000

TYPE I ERROR = 0.030 PROB(REJECT NULL HYP. GIVEN NULL HYP. TRUE)

TYPE II ERROR = 0.050 PROB(ACCEPT NULL HYP. GIVEN ALT. HYP. TRUE)

THETA	OPERATING CHARACTERISTIC VALUE			EXPECTED SAMPLE SIZE			VARIANCE S S
	APPROX.	EXACT	DIFFERENCE	APPROX.	EXACT	DIFFERENCE	EXACT
0.6389	0.9993	1.0000	0.0007	3.3948	5.0794	1.6846	12.65
0.8542	0.9922	0.9944	0.0022	6.1041	8.4198	2.3157	39.05
1.0000	0.9700	0.9784	0.0084	9.0358	12.0862	3.0504	94.33
1.0795	0.9425	0.9532	0.0107	11.1315	14.7478	3.6163	146.58
1.1634	0.8932	0.9098	0.0166	13.6999	18.5420	4.8421	240.34
1.2519	0.8111	0.8206	0.0095	16.5941	22.3180	5.7239	346.38
1.3450	0.6900	0.6856	-.0044	19.3657	25.9650	6.5993	457.10
1.3933	0.6166	0.6032	-.0134	20.4904	27.4370	6.9466	533.59
1.4933	0.4584	0.4226	-.0358	21.8123	28.6406	6.8283	494.17
1.5450	0.3813	0.3366	-.0447	21.9377	28.1196	6.1819	457.07
1.6519	0.2477	0.1964	-.0513	21.2399	26.5744	5.3345	367.64
1.7634	0.1507	0.1054	-.0453	19.7300	23.7762	4.0462	252.05
1.8795	0.0879	0.0534	-.0345	17.9461	20.8894	2.9433	154.67
2.0000	0.0500	0.0240	-.0260	16.2272	18.4440	2.2168	100.79
2.2542	0.0156	0.0064	-.0092	13.4454	14.8282	1.3828	45.19
2.7389	0.0020	0.0006	-.0014	10.4947	11.4204	0.9257	15.25

EXPONENTIAL SEQUENTIAL SAMPLING

F(X, T) = T * EXP(-T*X)

TEST: NULL HYPOTHESIS: THETA = 1.000

TEST: ALTERNATE HYPOTHESIS: THETA = 2.000

TYPE I ERROR = 0.050 PROB(REJECT NULL HYP. GIVEN NULL HYP. TRUE)

TYPE II ERROR = 0.030 PROB(ACCEPT NULL HYP. GIVEN ALT. HYP. TRUE)

THETA	OPERATING CHARACTERISTIC VALUE			EXPECTED SAMPLE SIZE			VARIANCE S S
	APPROX	EXACT	DIFFERENCE	APPROX.	EXACT	DIFFERENCE	EXACT
0.6389	0.9980	0.9992	0.0012	3.9473	5.6488	1.7015	14.26
0.8542	0.9844	0.9900	0.0056	7.0250	9.3360	2.3110	44.09
1.0000	0.9500	0.9654	0.0154	10.2142	13.3470	3.1328	103.10
1.0795	0.9121	0.9288	0.0167	12.3948	16.0870	3.6922	156.17
1.1634	0.8493	0.8712	0.0219	14.9499	19.7720	4.8221	257.56
1.2519	0.7523	0.7734	0.0211	17.6545	23.4264	5.7719	361.46
1.3450	0.6187	0.6210	0.0023	20.0000	26.0384	6.0384	444.44
1.3933	0.5416	0.5346	-.0070	20.8268	27.1352	6.3084	492.84
1.4933	0.3834	0.3562	-.0272	21.4601	27.9350	6.4749	486.82
1.5450	0.3100	0.2750	-.0350	21.2420	27.4202	6.1782	471.98
1.6519	0.1889	0.1496	-.0393	19.9642	24.6996	4.7354	357.84
1.7634	0.1068	0.0720	-.0348	18.0804	21.7782	3.6978	233.57
1.8795	0.0575	0.0306	-.0269	16.1170	18.8212	2.7042	153.23
2.0000	0.0300	0.0190	-.0110	14.3551	16.1102	1.7551	88.77
2.2542	0.0078	0.0036	-.0042	11.6829	12.9822	1.2993	40.26
2.7389	0.0007	0.0006	-.0001	9.0258	9.9114	0.8856	12.66

EXPONENTIAL SEQUENTIAL SAMPLING

F(X, T) = T * EXP(-T*X)

TEST: NULL HYPOTHESIS: THETA = 1.000

TEST: ALTERNATE HYPOTHESIS: THETA = 2.000

TYPE I ERROR = 0.050 PROB(REJECT NULL HYP. GIVEN NULL HYP. TRUE)

TYPE II ERROR = 0.050 PROB(ACCEPT NULL HYP. GIVEN ALT. HYP. TRUE)

THETA	OPERATING CHARACTERISTIC VALUE			EXPECTED SAMPLE SIZE			VARIANCE S S
	APPROX.	EXACT	DIFFERENCE	APPROX.	EXACT	DIFFERENCE	EXACT
0.5368	0.9994	1.0000	0.0006	2.5143	4.0232	1.5089	7.27
0.8542	0.9840	0.9920	0.0080	5.9687	8.1998	2.2311	38.06
1.0000	0.9500	0.9640	0.0140	8.6361	11.7038	3.0677	83.41
1.0795	0.9134	0.9314	0.0180	10.4374	14.1600	3.7226	131.68
1.1634	0.8540	0.8730	0.0190	12.5310	16.8628	4.3318	191.27
1.2519	0.7645	0.7806	0.0161	14.7496	20.1598	5.4102	267.85
1.3450	0.6431	0.6396	-.0035	16.7390	22.8174	6.0784	340.88
1.3933	0.5731	0.5602	-.0129	17.5034	24.4642	6.9608	401.87
1.4933	0.4269	0.3912	-.0357	18.3317	24.6248	6.2931	368.55
1.5459	0.3569	0.3122	-.0447	18.3607	24.2476	5.8869	338.41
1.6519	0.2355	0.1852	-.0503	17.7451	22.6040	4.8589	294.52
1.7634	0.1460	0.0996	-.0464	16.5377	20.4000	3.8623	195.97
1.8795	0.0866	0.0524	-.0342	15.1121	17.9822	2.8701	132.95
2.0000	0.0500	0.0244	-.0256	13.7201	15.9342	2.2141	88.13
2.2542	0.0159	0.0068	-.0091	11.4237	12.7886	1.3649	38.88
3.0368	0.0006	0.0	-.0006	8.0820	8.8998	0.8178	7.69

EXPONENTIAL SEQUENTIAL SAMPLING

F(X, T) = T * EXP(-T*X)

TEST: NULL HYPOTHESIS: THETA = 1.000

TEST: ALTERNATE HYPOTHESIS: THETA = 2.000

TYPE I ERROR = 0.100 PROB(REJECT NULL HYP. GIVEN NULL HYP. TRUE)

TYPE II ERROR = 0.010 PROB(ACCEPT NULL HYP. GIVEN ALT. HYP. TRUE)

THETA	OPERATING CHARACTERISTIC VALUE			EXPECTED SAMPLE SIZE			VARIANCE S S
	APPROX.	EXACT	DIFFERENCE	APPROX.	EXACT	DIFFERENCE	EXACT
0.5368	0.9968	0.9988	0.0020	3.8285	5.3152	1.4867	10.31
0.8542	0.9597	0.9758	0.0161	8.8489	11.2146	2.3657	53.92
1.0000	0.9000	0.9230	0.0230	12.4508	15.5058	3.0550	111.00
1.0795	0.8439	0.8736	0.0297	14.7479	18.6784	3.9305	185.53
1.1634	0.7602	0.7918	0.0316	17.2558	21.8034	4.5476	275.00
1.2519	0.6428	0.6710	0.0282	19.6295	25.2048	5.5753	406.43
1.3450	0.4950	0.5096	0.0146	21.2488	26.9410	5.6922	528.64
1.3933	0.4156	0.4242	0.0086	21.5610	27.9904	6.4294	610.05
1.4933	0.2650	0.2558	-.0092	20.9853	26.8316	5.8463	560.74
1.5450	0.2013	0.1856	-.0157	20.1625	25.4454	5.2829	498.43
1.6519	0.1063	0.0856	-.0207	17.8922	22.0548	4.1626	388.35
1.7634	0.0511	0.0394	-.0117	15.4317	18.0952	2.6635	215.25
1.8795	0.0231	0.0158	-.0073	13.2595	15.3056	2.0461	128.46
2.0000	0.0100	0.0062	-.0038	11.5177	13.1950	1.6773	81.71
2.2542	0.0018	0.0016	-.0002	9.1396	10.3220	1.1824	33.74
3.0368	0.0	0.0	0.0	6.3004	7.0918	0.7914	6.03

EXPONENTIAL SEQUENTIAL SAMPLING

F(X, T) = T * EXP(-T*X)

TEST: NULL HYPOTHESIS: THETA = 1.000

TEST: ALTERNATE HYPOTHESIS: THETA = 2.000

TYPE I ERROR = 0.100 PROB(REJECT NULL HYP. GIVEN NULL HYP. TRUE)

TYPE II ERROR = 0.100 PROB(ACCEPT NULL HYP. GIVEN ALT. HYP. TRUE)

THETA	OPERATING CHARACTERISTIC VALUE			EXPECTED SAMPLE SIZE			VARIANCE S S
	APPROX	EXACT	DIFFERENCE	APPROX.	EXACT	DIFFERENCE	EXACT
0.5368	0.9959	0.9986	0.0027	1.8632	3.4116	1.5484	5.60
0.8542	0.9559	0.9734	0.0175	4.1949	6.5974	2.4025	27.16
1.0000	0.9000	0.9242	0.0242	5.7284	8.8190	3.0906	51.57
1.0795	0.8529	0.8776	0.0247	6.6498	10.1572	3.5074	69.41
1.1634	0.7889	0.8150	0.0261	7.6306	11.8284	4.1978	97.40
1.2519	0.7066	0.7176	0.0110	8.5953	13.0708	4.4755	117.45
1.3450	0.6081	0.6004	-.0077	9.4377	14.1906	4.7529	131.24
1.3933	0.5547	0.5372	-.0175	9.7778	14.8288	5.0510	140.42
1.4933	0.4453	0.4036	-.0417	10.2405	14.9176	4.6771	133.84
1.5450	0.3919	0.3406	-.0513	10.3521	15.1678	4.8157	134.46
1.6519	0.2934	0.2338	-.0596	10.3409	14.4234	4.0825	114.55
1.7634	0.2111	0.1494	-.0617	10.0704	13.6414	3.5710	91.95
1.8795	0.1471	0.0950	-.0521	9.6281	12.4552	2.8271	65.58
2.0000	0.1000	0.0538	-.0462	9.1007	11.4688	2.3681	48.87
2.1890	0.0544	0.0254	-.0290	8.2869	9.9982	1.7113	30.62
3.0368	0.0041	0.0006	-.0035	5.9892	6.7952	0.8060	5.92

EXPONENTIAL SEQUENTIAL SAMPLING

F(X, T) = T * EXP(-T*X)

TEST: NULL HYPOTHESIS: THETA = 1.000

TEST: ALTERNATE HYPOTHESIS: THETA = 3.000

TYPE I ERROR = 0.100 PROB(REJECT NULL HYP. GIVEN NULL HYP. TRUE)

TYPE II ERROR = 0.010 PROB(ACCEPT NULL HYP. GIVEN ALT. HYP. TRUE)

THETA	OPERATING CHARACTERISTIC VALUE			EXPECTED SAMPLE SIZE			VARIANCE S S
	APPROX.	EXACT	DIFFERENCE	APPROX.	EXACT	DIFFERENCE	EXACT
0.5000	0.9898	0.9968	0.0070	1.5270	2.8682	1.3412	3.22
0.7660	0.9597	0.9770	0.0173	2.7941	4.4440	1.6499	8.95
1.0000	0.9000	0.9328	0.0328	4.2385	6.3568	2.1183	20.44
1.1362	0.8439	0.8892	0.0453	5.1984	7.8080	2.6096	32.21
1.2859	0.7602	0.8118	0.0516	6.2866	9.3880	3.1014	50.83
1.4497	0.6428	0.6956	0.0528	7.3783	11.0616	3.6833	80.19
1.6278	0.4950	0.5252	0.0302	8.2268	12.4974	4.2706	109.53
1.7223	0.4156	0.4282	0.0126	8.4660	12.5690	4.1030	111.57
1.9223	0.2650	0.2520	-.0130	8.4657	12.2188	3.7531	110.66
2.0278	0.2013	0.1772	-.0241	8.2395	11.8244	3.5849	104.08
2.2497	0.1063	0.0780	-.0283	7.4944	10.1602	2.6658	71.05
2.4859	0.0511	0.0308	-.0203	6.6154	8.4684	1.8530	39.84
2.7362	0.0231	0.0122	-.0109	5.8093	7.1912	1.3819	24.34
3.0000	0.0100	0.0038	-.0062	5.1502	6.2770	1.1268	14.62
3.4199	0.0027	0.0008	-.0019	4.4258	5.3106	0.8848	7.20
4.5000	0.0001	0.0	-.0001	3.5032	4.1964	0.6932	2.11

EXPONENTIAL SEQUENTIAL SAMPLING

F(X, T) = T * EXP(-T*X)

TEST: NULL HYPOTHESIS: THETA = 1.000

TEST: ALTERNATE HYPOTHESIS: THETA = 3.000

TYPE I ERROR = 0.100 PROB(REJECT NULL HYP. GIVEN NULL HYP. TRUE)

TYPE II ERROR = 0.100 PROB(ACCEPT NULL HYP. GIVEN ALT. HYP. TRUE)

THETA	OPERATING CHARACTERISTIC VALUE			EXPECTED SAMPLE SIZE			VARIANCE S S
	APPROX.	EXACT	DIFFERENCE	APPROX.	EXACT	DIFFERENCE	EXACT
0.5000	0.9878	0.9962	0.0084	0.7388	2.0868	1.3480	1.87
0.8199	0.9456	0.9714	0.0258	1.4607	3.2350	1.7743	5.61
1.0000	0.9000	0.9328	0.0328	1.9501	4.0352	2.0851	9.37
1.1362	0.8529	0.8904	0.0375	2.3440	4.7664	2.4224	14.23
1.2859	0.7889	0.8226	0.0337	2.7799	5.4994	2.7195	19.51
1.4497	0.7066	0.7232	0.0166	3.2308	6.3842	3.1534	25.98
1.6278	0.6081	0.5944	-.0137	3.6539	7.0598	3.4059	31.21
1.7223	0.5547	0.5246	-.0301	3.8393	7.2222	3.3829	31.90
1.9223	0.4453	0.3778	-.0675	4.1312	7.5544	3.4232	32.15
2.0278	0.3919	0.3162	-.0757	4.2304	7.4108	3.1804	29.52
2.2497	0.2934	0.1976	-.0958	4.3314	7.2808	2.9494	26.27
2.4859	0.2111	0.1198	-.0913	4.3171	6.6896	2.3725	18.84
2.7362	0.1471	0.0700	-.0771	4.2183	6.1716	1.9533	13.37
3.0000	0.1000	0.0382	-.0618	4.0694	5.6578	1.5884	9.61
3.4199	0.0544	0.0152	-.0392	3.8116	4.9962	1.1846	5.69
4.5000	0.0122	0.0022	-.0100	3.2769	4.0524	0.7755	1.89

EXPONENTIAL SEQUENTIAL SAMPLING

F(X, T) = T * EXP(-T*X)

TEST: NULL HYPOTHESIS: THETA = 1.000

TEST: ALTERNATE HYPOTHESIS: THETA = 5.000

TYPE I ERROR = 0.030 PROB(REJECT NULL HYP. GIVEN NULL HYP. TRUE)

TYPE II ERROR = 0.050 PROB(ACCEPT NULL HYP. GIVEN ALT. HYP. TRUE)

THETA	OPERATING CHARACTERISTIC VALUE			EXPECTED SAMPLE SIZE			VARIANCE S S
	APPROX.	EXACT	DIFFERENCE	APPROX.	EXACT	DIFFERENCE	EXACT
0.2060	0.9997	0.9998	0.0001	0.1665	1.2402	1.0737	0.27
0.6574	0.9922	0.9962	0.0040	0.6514	1.9802	1.3288	1.63
1.0000	0.9700	0.9858	0.0158	1.1598	2.7998	1.6400	4.18
1.2196	0.9425	0.9650	0.0225	1.5542	3.4544	1.9002	6.71
1.4755	0.8932	0.9202	0.0270	2.0695	4.3000	2.2305	11.45
1.7706	0.8111	0.8358	0.0247	2.6977	5.4852	2.7875	19.96
2.1068	0.6900	0.6834	-.0066	3.3713	6.7522	3.3809	28.58
2.2907	0.6166	0.5862	-.0304	3.6843	7.2278	3.5435	33.38
2.6907	0.4584	0.3734	-.0850	4.1689	7.6500	3.4811	33.35
2.9068	0.3813	0.2830	-.0983	4.3154	7.4602	3.1448	27.78
3.3706	0.2477	0.1410	-.1067	4.4114	7.0262	2.6148	20.09
3.8755	0.1507	0.0660	-.0847	4.3085	6.2050	1.8965	12.16
4.4196	0.0879	0.0286	-.0593	4.1041	5.5068	1.4027	7.11
5.0000	0.0500	0.0130	-.0370	3.8721	4.9406	1.0685	4.39
6.2574	0.0156	0.0020	-.0136	3.4580	4.2206	0.7626	1.90
9.8060	0.0008	0.0	-.0008	2.8714	3.4342	0.5628	0.45

EXPONENTIAL SEQUENTIAL SAMPLING

F(X, T) = T * EXP(-T*X)

TEST: NULL HYPOTHESIS: THETA = 1.000

TEST: ALTERNATE HYPOTHESIS: THETA = 5.000

TYPE I ERROR = 0.050 PROB(REJECT NULL HYP. GIVEN NULL HYP. TRUE)

TYPE II ERROR = 0.030 PROB(ACCEPT NULL HYP. GIVEN ALT. HYP. TRUE)

THETA	OPERATING CHARACTERISTIC VALUE			EXPECTED SAMPLE SIZE			VARIANCE S S
	APPROX.	EXACT	DIFFERENCE	APPROX.	EXACT	DIFFERENCE	EXACT
0.2060	0.9992	0.9998	0.0006	0.1938	1.2690	1.0752	0.31
0.6574	0.9844	0.9956	0.0112	0.7497	2.0776	1.3279	1.81
1.0000	0.9500	0.9764	0.0264	1.3111	2.9448	1.6337	4.31
1.2196	0.9121	0.9460	0.0339	1.7306	3.6214	1.8908	6.90
1.4755	0.8493	0.8918	0.0425	2.2583	4.4922	2.2339	11.81
1.7706	0.7523	0.7930	0.0407	2.8701	5.6084	2.7383	19.94
2.1068	0.6187	0.6288	0.0101	3.4817	6.7560	3.2743	28.11
2.2907	0.5416	0.5292	-.0124	3.7448	7.0030	3.2582	32.00
2.6907	0.3834	0.3206	-.0628	4.1015	7.3366	3.2351	31.16
2.9068	0.3100	0.2382	-.0718	4.1785	7.0478	2.8693	25.62
3.3706	0.1889	0.1134	-.0755	4.1464	6.3878	2.2414	18.34
3.8755	0.1068	0.0474	-.0594	3.9482	5.6030	1.6548	11.25
4.4196	0.0575	0.0170	-.0405	3.6858	4.9658	1.2800	6.81
5.0000	0.0300	0.0070	-.0230	3.4254	4.4194	0.9940	4.03
6.2574	0.0078	0.0016	-.0062	3.0047	3.7434	0.7387	1.64
9.8060	0.0002	0.0	-.0002	2.4666	3.0774	0.6108	0.41

EXPONENTIAL SEQUENTIAL SAMPLING

F(X, T) = T * EXP(-T*X)

TEST: NULL HYPOTHESIS: THETA = 1.000

TEST: ALTERNATE HYPOTHESIS: THETA = 5.000

TYPE I ERROR = 0.050 PROB(REJECT NULL HYP. GIVEN NULL HYP. TRUE)

TYPE II ERROR = 0.050 PROB(ACCEPT NULL HYP. GIVEN ALT. HYP. TRUE)

THETA	OPERATING CHARACTERISTIC VALUE			EXPECTED SAMPLE SIZE			VARIANCE S S
	APPROX	EXACT	DIFFERENCE	APPROX.	EXACT	DIFFERENCE	EXACT
0.2060	0.9991	0.9998	0.0007	0.1651	1.2390	1.0739	0.27
0.7221	0.9787	0.9928	0.0141	0.7313	2.1120	1.3807	1.99
1.0000	0.9500	0.9744	0.0244	1.1085	2.7488	1.6403	2.90
1.2196	0.9134	0.9450	0.0316	1.4573	3.3300	1.8727	5.93
1.4755	0.8540	0.8908	0.0368	1.8929	4.0944	2.2015	9.74
1.7706	0.7645	0.7972	0.0327	2.3979	5.0924	2.6945	16.87
2.1063	0.6431	0.6442	0.0011	2.9140	6.0434	3.1294	21.98
2.2907	0.5731	0.5492	-.0239	3.1472	6.3204	3.1732	25.32
2.6907	0.4269	0.3484	-.0785	3.5036	6.6712	3.1676	23.81
2.9068	0.3569	0.2662	-.0907	3.6118	6.5186	2.9068	21.20
3.3706	0.2355	0.1370	-.0985	3.6955	6.0549	2.3693	15.98
3.8755	0.1460	0.0620	-.0840	3.6113	5.4184	1.8071	10.18
4.4195	0.0866	0.0266	-.0600	3.4560	4.8400	1.3840	6.25
5.0000	0.0500	0.0130	-.0370	3.2739	4.3382	1.0643	3.75
5.9321	0.0213	0.0034	-.0179	3.0146	3.8418	0.8272	2.06
9.8060	0.0009	0.0002	-.0007	2.4464	3.0566	0.6102	0.41

NORMAL SEQUENTIAL SAMPLING

TEST: NULL HYPOTHESIS: THETA = 0.0

TEST: ALTERNATE HYPOTHESIS: THETA = 0.500

TYPE I ERROR = 0.010 PROB(REJECT NULL HYP. GIVEN NULL HYP. TRUE)

TYPE II ERROR = 0.010 PROB(ACCEPT NULL HYP. GIVEN ALT. HYP. TRUE)

STANDARD DEVIATION = 1.0

THETA	OPERATING CHARACTERISTIC VALUE			EXPECTED SAMPLE SIZE			VARIANCE S S
	APPROX	EXACT	DIFFERENCE	APPROX.	EXACT	DIFFERENCE	EXACT
-.2000	0.9997	1.0000	0.0003	20.4123	22.0326	1.6203	108.45
0.0	0.9900	0.9924	0.0024	36.0258	38.6030	2.5772	547.12
0.1000	0.9403	0.9494	0.0091	53.9544	58.7520	4.7976	1850.48
0.1500	0.8627	0.8802	0.0175	66.6694	74.0776	7.4082	3186.48
0.1750	0.7988	0.8112	0.0124	73.2172	80.8372	7.6200	4082.74
0.2000	0.7148	0.7290	0.0142	78.9785	88.0358	9.0573	4884.25
0.2250	0.6129	0.6212	0.0083	83.0049	93.7790	10.7741	5690.77
0.2500	0.5000	0.5010	0.0010	84.4606	95.2400	10.7794	6025.12
0.2750	0.3871	0.3842	-.0029	83.0050	92.3230	9.3180	5574.62
0.3000	0.2852	0.2770	-.0082	78.9783	88.7704	9.7921	4801.81
0.3250	0.2012	0.1882	-.0130	73.2169	82.0806	8.8637	4320.76
0.3500	0.1373	0.1262	-.0111	66.6691	73.5642	6.8951	3226.14
0.4000	0.0597	0.0546	-.0051	53.9541	58.3528	4.3987	1771.29
0.5000	0.0100	0.0090	-.0010	36.0257	38.7322	2.7065	574.71
0.6000	0.0016	0.0016	0.0	26.1735	28.0710	1.8975	225.05

NORMAL SEQUENTIAL SAMPLING

TEST: NULL HYPOTHESIS: THETA = 0.0

TEST: ALTERNATE HYPOTHESIS: THETA = 0.500

TYPE I ERROR = 0.010 PROB(REJECT NULL HYP. GIVEN NULL HYP. TRUE)

TYPE II ERROR = 0.100 PROB(ACCEPT NULL HYP. GIVEN ALT. HYP. TRUE)

STANDARD DEVIATION = 1.0

THETA	OPERATING CHARACTERISTIC VALUE			EXPECTED SAMPLE SIZE			VARIANCE S S
	APPROX.	EXACT	DIFFERENCE	APPROX.	EXACT	DIFFERENCE	EXACT
-.1000	0.9982	0.9988	0.0006	13.0318	15.0312	1.9994	118.01
0.0	0.9900	0.9928	0.0028	17.7969	20.5672	2.7703	296.55
0.1000	0.9489	0.9536	0.0047	25.9397	30.1534	4.2137	792.73
0.1625	0.8741	0.8816	0.0075	32.8535	38.9692	6.1157	1338.63
0.2000	0.7987	0.8030	0.0043	37.0183	44.9988	7.9805	1751.93
0.2250	0.7350	0.7336	-.0014	39.4144	48.0618	8.6474	1948.90
0.2500	0.6625	0.6528	-.0097	41.2639	49.8762	8.6123	2069.64
0.2750	0.5844	0.5684	-.0160	42.4118	51.2508	8.8390	1978.18
0.3000	0.5050	0.4822	-.0228	42.7925	51.0280	8.2355	1843.56
0.3250	0.4282	0.3990	-.0292	42.4397	50.4524	8.0127	1651.23
0.3500	0.3572	0.3286	-.0286	41.4664	48.0750	6.6086	1368.13
0.3750	0.2942	0.2602	-.0340	40.0267	46.3478	6.3211	1135.03
0.4000	0.2398	0.2066	-.0332	38.2805	43.9628	5.6823	964.31
0.4500	0.1561	0.1292	-.0269	34.3963	38.7804	4.3841	609.94
0.5000	0.1000	0.0778	-.0222	30.5646	33.9746	3.4100	416.49
0.6000	0.0403	0.0270	-.0133	24.1489	26.4948	2.3459	195.92

NORMAL SEQUENTIAL SAMPLING

TEST: NULL HYPOTHESIS: THETA = 0.0

TEST: ALTERNATE HYPOTHESIS: THETA = 0.500

TYPE I ERROR = 0.100 PROB(REJECT NULL HYP. GIVEN NULL HYP. TRUE)

TYPE II ERROR = 0.010 PROB(ACCEPT NULL HYP. GIVEN ALT. HYP. TRUE)

STANDARD DEVIATION = 1.0

THETA	OPERATING CHARACTERISTIC VALUE			EXPECTED SAMPLE SIZE			VARIANCE S S
	APPROX.	EXACT	DIFFERENCE	APPROX.	EXACT	DIFFERENCE	EXACT
-.2000	0.9839	0.9910	0.0071	19.5121	21.3240	1.8119	102.56
0.0	0.9000	0.9228	0.0228	30.5647	33.9802	3.4155	420.04
0.0750	0.8060	0.8462	0.0402	36.3673	41.8970	5.5297	831.06
0.1000	0.7602	0.8006	0.0404	38.2805	44.4060	6.1255	972.67
0.1250	0.7058	0.7464	0.0406	40.0269	46.6476	6.6207	1219.92
0.1500	0.6428	0.6842	0.0414	41.4664	49.1252	7.6588	1418.33
0.1750	0.5718	0.6054	0.0336	42.4398	49.8308	7.3910	1509.13
0.2000	0.4950	0.5314	0.0364	42.7924	52.0590	9.2666	1845.59
0.2250	0.4156	0.4406	0.0250	42.4115	50.7560	8.3445	1891.03
0.2500	0.3375	0.3556	0.0181	41.2639	50.4324	9.1685	1988.87
0.2750	0.2650	0.2720	0.0070	39.4143	47.6124	8.1981	1918.93
0.3000	0.2013	0.2006	-.0007	37.0182	45.0462	8.0280	1729.79
0.3250	0.1482	0.1414	-.0068	34.2821	41.4492	7.1671	1481.87
0.3500	0.1063	0.1012	-.0051	31.4161	36.9458	5.5297	1150.99
0.4000	0.0511	0.0454	-.0057	25.9396	30.2384	4.2988	731.92
0.4500	0.0231	0.0204	-.0027	21.3588	24.5196	3.1608	443.06
0.5000	0.0100	0.0088	-.0012	17.7969	20.2142	2.4173	278.40
0.6000	0.0018	0.0016	-.0002	13.0318	14.8910	1.8592	117.03

NORMAL SEQUENTIAL SAMPLING

TEST: NULL HYPOTHESIS: THETA = 0.0

TEST: ALTERNATE HYPOTHESIS: THETA = 0.500

TYPE I ERROR = 0.100 PROB(REJECT NULL HYP. GIVEN NULL HYP. TRUE)

TYPE II ERROR = 0.100 PROB(ACCEPT NULL HYP. GIVEN ALT. HYP. TRUE)

STANDARD DEVIATION = 1.0

THETA	OPERATING CHARACTERISTIC VALUE			EXPECTED SAMPLE SIZE			VARIANCE S S
	APPROX.	EXACT	DIFFERENCE	APPROX.	EXACT	DIFFERENCE	EXACT
-.2000	0.9812	0.9892	0.0080	9.3983	11.0712	1.6729	47.79
-.1000	0.9559	0.9692	0.0133	11.4481	13.6364	2.1883	87.67
0.0	0.9000	0.9228	0.0228	14.0623	17.0822	3.0199	157.50
0.0500	0.8529	0.8806	0.0277	15.5094	19.2306	3.7212	212.64
0.1000	0.7889	0.8154	0.0265	16.9277	21.2806	4.3529	273.41
0.1250	0.7500	0.7768	0.0268	17.5778	22.2658	4.6880	303.25
0.1500	0.7066	0.7336	0.0270	18.1572	23.4664	5.3092	359.31
0.1750	0.6591	0.6766	0.0175	18.6411	23.6200	4.9789	353.60
0.2000	0.6081	0.6188	0.0107	19.0063	24.2254	5.2191	374.72
0.2250	0.5547	0.5610	0.0063	19.2339	24.7586	5.5247	386.95
0.2500	0.5000	0.5010	0.0010	19.3112	25.0032	5.6920	404.36
0.2750	0.4453	0.4380	-.0073	19.2338	24.6106	5.3768	410.61
0.3000	0.3919	0.3778	-.0141	19.0063	24.2920	5.2857	386.85
0.3250	0.3409	0.3210	-.0199	18.6411	23.8398	5.1987	360.91
0.3500	0.2934	0.2698	-.0236	18.1572	23.1894	5.0322	348.17
0.3750	0.2500	0.2210	-.0290	17.5778	22.4296	4.8518	304.94
0.4000	0.2111	0.1796	-.0315	16.9277	21.4452	4.5175	284.93
0.4500	0.1471	0.1192	-.0279	15.5094	19.0848	3.5754	203.47
0.5000	0.1000	0.0778	-.0222	14.0622	17.0184	2.9562	150.55
0.6000	0.0441	0.0290	-.0151	11.4481	13.5858	2.1377	87.02

NORMAL SEQUENTIAL SAMPLING

TEST: NULL HYPOTHESIS: THETA = 0.0

TEST: ALTERNATE HYPOTHESIS: THETA = 1.000

TYPE I ERROR = 0.001 PROB(REJECT NULL HYP. GIVEN NULL HYP. TRUE)

TYPE II ERROR = 0.001 PROB(ACCEPT NULL HYP. GIVEN ALT. HYP. TRUE)

STANDARD DEVIATION = 1.0

THETA	OPERATING CHARACTERISTIC VALUE			EXPECTED SAMPLE SIZE			VARIANCE S S
	APPROX.	EXACT	DIFFERENCE	APPROX.	EXACT	DIFFERENCE	EXACT
-.2000	0.9999	1.0000	0.0001	9.8656	10.9586	1.0930	21.76
0.0	0.9990	0.9996	0.0006	13.7859	15.2438	1.4579	60.40
0.2000	0.9844	0.9902	0.0058	22.3037	24.7244	2.4207	248.78
0.3500	0.8882	0.9036	0.0154	35.7452	40.2964	4.5512	927.41
0.4000	0.7992	0.8196	0.0204	41.3312	47.6670	6.3358	1411.04
0.4500	0.6661	0.6830	0.0169	45.8933	53.7954	7.9021	1846.55
0.4750	0.5855	0.5936	0.0081	47.2346	55.3940	8.1594	2036.23
0.5000	0.5000	0.5004	0.0004	47.7033	56.4458	8.7425	2088.05
0.5250	0.4145	0.4080	-.0065	47.2347	55.5228	8.2881	2029.92
0.5500	0.3339	0.3250	-.0089	45.8932	53.7954	7.9022	1878.61
0.6000	0.2008	0.1806	-.0202	41.3311	48.3298	6.9987	1448.90
0.6500	0.1118	0.1008	-.0110	35.7451	40.3280	4.5829	872.39
0.8000	0.0156	0.0112	-.0044	22.3037	24.5970	2.2933	248.80
1.0000	0.0010	0.0006	-.0004	13.7859	15.1722	1.3863	60.18
1.2000	0.0001	0.0	-.0001	9.8656	10.9526	1.0870	21.42

NORMAL SEQUENTIAL SAMPLING

TEST: NULL HYPOTHESIS: THETA = 0.0

TEST: ALTERNATE HYPOTHESIS: THETA = 1.000

TYPE I ERROR = 0.001 PROB(REJECT NULL HYP. GIVEN NULL HYP. TRUE)

TYPE II ERROR = 0.010 PROB(ACCEPT NULL HYP. GIVEN ALT. HYP. TRUE)

STANDARD DEVIATION = 1.0

THETA	OPERATING CHARACTERISTIC VALUE			EXPECTED SAMPLE SIZE			VARIANCE S S
	APPROX	EXACT	DIFFERENCE	APPROX.	EXACT	DIFFERENCE	EXACT
-.2000	0.9999	0.9998	-.0001	6.5763	7.7006	1.1243	14.53
0.0	0.9990	0.9996	0.0006	9.1853	10.5934	1.4081	40.90
0.2000	0.9850	0.9914	0.0064	14.7740	17.2408	2.4668	167.99
0.3500	0.9024	0.9146	0.0122	23.2073	27.5808	4.3735	539.09
0.4000	0.8317	0.8472	0.0155	26.6789	32.7570	6.0781	791.43
0.4500	0.7291	0.7342	0.0051	29.7748	36.5752	6.8004	1037.10
0.4750	0.6670	0.6688	0.0018	30.9483	37.9596	7.0113	1061.37
0.5000	0.5997	0.5924	-.0073	31.7582	38.8568	7.0986	1064.51
0.5250	0.5298	0.5156	-.0142	32.1510	39.5370	7.3860	1127.53
0.5500	0.4601	0.4392	-.0209	32.1137	39.2140	7.1003	1019.78
0.6000	0.3312	0.2988	-.0324	30.8881	37.6204	6.7323	835.37
0.6500	0.2267	0.1982	-.0285	28.5992	33.3192	4.7200	574.48
0.7000	0.1500	0.1242	-.0258	25.8612	29.8424	3.9812	396.65
0.8000	0.0622	0.0420	-.0202	20.6080	23.4054	2.7974	201.93
1.0000	0.0100	0.0046	-.0054	13.5654	15.0510	1.4856	57.71
1.2000	0.0016	0.0008	-.0008	9.8278	10.9326	1.1048	21.95

NORMAL SEQUENTIAL SAMPLING

TEST: NULL HYPOTHESIS: THETA = 0.0

TEST: ALTERNATE HYPOTHESIS: THETA = 1.000

TYPE I ERROR = 0.001 PROB(REJECT NULL HYP. GIVEN NULL HYP. TRUE)

TYPE II ERROR = 0.100 PROB(ACCEPT NULL HYP. GIVEN ALT. HYP. TRUE)

STANDARD DEVIATION = 1.0

THETA	OPERATING CHARACTERISTIC VALUE			EXPECTED SAMPLE SIZE			VARIANCE S S
	APPROX	EXACT	DIFFERENCE	APPROX.	EXACT	DIFFERENCE	EXACT
-.2000	0.9999	0.9998	-.0001	3.2871	4.3620	1.0749	8.40
0.0	0.9990	0.9994	0.0004	4.5850	6.0320	1.4470	22.41
0.2000	0.9873	0.9920	0.0047	7.2867	9.5858	2.2991	85.70
0.4000	0.8871	0.8866	-.0005	12.7354	17.6536	4.9182	338.10
0.4500	0.8258	0.8088	-.0170	14.3063	19.7782	5.4719	421.01
0.5000	0.7472	0.7164	-.0308	15.6563	21.5674	5.9111	444.85
0.5500	0.6560	0.6064	-.0496	16.6054	23.1920	6.5866	471.74
0.6000	0.5598	0.4964	-.0634	17.0581	23.0282	5.9701	403.22
0.6500	0.4666	0.3948	-.0718	17.0304	22.4224	5.3920	329.62
0.7000	0.3821	0.3092	-.0729	16.6202	21.2024	4.5822	248.88
0.7500	0.3091	0.2330	-.0761	15.9534	20.0172	4.0638	198.32
0.8000	0.2481	0.1700	-.0781	15.1442	18.9576	3.8134	154.63
0.9000	0.1580	0.0900	-.0680	13.4091	16.3794	2.9703	91.67
1.0000	0.1000	0.0574	-.0426	11.7840	13.8396	2.0556	50.15
1.2000	0.0399	0.0158	-.0241	9.1993	10.6012	1.4019	20.29

NORMAL SEQUENTIAL SAMPLING

TEST: NULL HYPOTHESIS: THETA = 0.0

TEST: ALTERNATE HYPOTHESIS: THETA = 1.000

TYPE I ERROR = 0.001 PROB(REJECT NULL HYP. GIVEN NULL HYP. TRUE)

TYPE II ERROR = 0.200 PROB(ACCEPT NULL HYP. GIVEN ALT. HYP. TRUE)

STANDARD DEVIATION = 1.0

THETA	OPERATING CHARACTERISTIC VALUE			EXPECTED SAMPLE SIZE			VARIANCE S S
	APPROX.	EXACT	DIFFERENCE	APPROX.	EXACT	DIFFERENCE	EXACT
-.2000	0.9999	0.9998	-.0001	2.2969	3.3630	1.0661	6.07
0.0	0.9990	0.9994	0.0004	3.2003	4.6098	1.4095	16.33
0.2500	0.9801	0.9858	0.0057	5.7753	8.4846	2.7093	98.70
0.4000	0.9108	0.9032	-.0076	8.6836	12.9056	4.2220	214.47
0.5000	0.8060	0.7680	-.0380	10.7518	16.1828	5.4310	315.49
0.5500	0.7364	0.6764	-.0600	11.5514	17.4714	5.9200	342.84
0.6000	0.6602	0.5784	-.0818	12.0928	17.9040	5.8112	292.42
0.6500	0.5825	0.4854	-.0971	12.3575	17.9016	5.5441	255.11
0.7000	0.5077	0.3992	-.1085	12.3725	17.5808	5.2083	193.69
0.7500	0.4386	0.3338	-.1048	12.1906	16.6736	4.4830	168.58
0.8000	0.3767	0.2666	-.1101	11.8699	16.0286	4.1587	128.83
0.9000	0.2752	0.1708	-.1044	11.0056	14.3110	3.3054	80.76
1.0000	0.2000	0.1118	-.0882	10.0520	12.6788	2.6268	49.03
1.2000	0.1052	0.0456	-.0596	8.3031	10.0306	1.7275	20.33
1.4000	0.0553	0.0176	-.0377	6.9179	8.1780	1.2601	9.91

NORMAL SEQUENTIAL SAMPLING

TEST: NULL HYPOTHESIS: THETA = 0.0

TEST: ALTERNATE HYPOTHESIS: THETA = 1.000

TYPE I ERROR = 0.010 PROB(REJECT NULL HYP. GIVEN NULL HYP. TRUE)

TYPE II ERROR = 0.001 PROB(ACCEPT NULL HYP. GIVEN ALT. HYP. TRUE)

STANDARD DEVIATION = 1.0

THETA	OPERATING CHARACTERISTIC VALUE			EXPECTED SAMPLE SIZE			VARIANCE S S
	APPROX.	EXACT	DIFFERENCE	APPROX.	EXACT	DIFFERENCE	EXACT
-.2000	0.9984	0.9996	0.0012	9.8278	10.9406	1.1128	21.73
0.0	0.9900	0.9964	0.0064	13.5654	15.1384	1.5730	59.24
0.2000	0.9378	0.9530	0.0152	20.6080	23.2080	2.6000	193.59
0.3000	0.8500	0.8774	0.0274	25.8613	29.9038	4.0425	448.48
0.3500	0.7733	0.8090	0.0357	28.5992	33.9508	5.3516	622.29
0.4000	0.6688	0.6994	0.0306	30.8881	36.8942	6.0061	831.29
0.4500	0.5399	0.5688	0.0289	32.1136	39.6712	7.5576	1065.71
0.5000	0.4003	0.4120	0.0117	31.7582	39.5800	7.8218	1109.69
0.5500	0.2709	0.2642	-.0067	29.7748	36.7528	6.9780	993.58
0.6000	0.1683	0.1572	-.0111	26.6789	32.2078	5.5289	755.83
0.7000	0.0538	0.0470	-.0068	19.9242	23.1364	3.2122	345.21
0.8000	0.0150	0.0114	-.0036	14.7739	17.0140	2.2401	164.36
1.0000	0.0010	0.0006	-.0004	9.1853	10.6276	1.4423	41.80
1.2000	0.0001	0.0	-.0001	6.5764	7.6772	1.1008	14.72

NORMAL SEQUENTIAL SAMPLING

TEST: NULL HYPOTHESIS: THETA = 0.0

TEST: ALTERNATE HYPOTHESIS: THETA = 1.000

TYPE I ERROR = 0.010 PROB(REJECT NULL HYP. GIVEN NULL HYP. TRUE)

TYPE II ERROR = 0.010 PROB(ACCEPT NULL HYP. GIVEN ALT. HYP. TRUE)

STANDARD DEVIATION = 1.0

THETA	OPERATING CHARACTERISTIC VALUE			EXPECTED SAMPLE SIZE			VARIANCE S S
	APPROX.	EXACT	DIFFERENCE	APPROX.	EXACT	DIFFERENCE	EXACT
-.2000	0.9984	0.9992	0.0008	6.5434	7.6878	1.1444	14.46
0.0	0.9900	0.9968	0.0068	9.0064	10.5066	1.5002	39.42
0.2000	0.9403	0.9562	0.0159	13.4886	16.0644	2.5758	131.63
0.3000	0.8627	0.8884	0.0257	16.6673	20.5198	3.8525	235.34
0.3500	0.7988	0.8260	0.0272	18.3043	22.6432	4.3389	307.51
0.4000	0.7148	0.7374	0.0226	19.7446	24.7170	4.9724	394.42
0.4500	0.6129	0.6280	0.0151	20.7514	26.5614	5.8100	466.75
0.5000	0.5000	0.4992	-.0008	21.1151	26.7970	5.6819	478.86
0.5500	0.3871	0.3694	-.0177	20.7512	26.9740	6.2228	481.18
0.6000	0.2852	0.2608	-.0244	19.7446	25.0036	5.2590	403.29
0.6500	0.2012	0.1746	-.0266	18.3042	22.7148	4.4106	318.81
0.7500	0.0913	0.0682	-.0231	15.0233	18.0166	2.9933	162.34
0.8000	0.0597	0.0428	-.0169	13.4886	16.0030	2.5144	127.07
1.0000	0.0100	0.0056	-.0044	9.0064	10.5188	1.5124	39.95
1.2000	0.0016	0.0002	-.0014	6.5434	7.6608	1.1174	14.65

NORMAL SEQUENTIAL SAMPLING

TEST: NULL HYPOTHESIS: THETA = 0.0

TEST: ALTERNATE HYPOTHESIS: THETA = 1.000

TYPE I ERROR = 0.010 PROB(REJECT NULL HYP. GIVEN NULL HYP. TRUE)

TYPE II ERROR = 0.100 PROB(ACCEPT NULL HYP. GIVEN ALT. HYP. TRUE)

STANDARD DEVIATION = 1.0

THETA	OPERATING CHARACTERISTIC VALUE			EXPECTED SAMPLE SIZE			VARIANCE S S
	APPROX.	EXACT	DIFFERENCE	APPROX.	EXACT	DIFFERENCE	EXACT
-.2000	0.9982	0.9992	0.0010	3.2579	4.3470	1.0891	8.28
0.0	0.9900	0.9938	0.0038	4.4492	5.9366	1.4874	19.99
0.2000	0.9489	0.9630	0.0141	6.4849	8.8822	2.3973	62.27
0.3000	0.8937	0.9104	0.0167	7.8541	10.9760	3.1219	97.30
0.3500	0.8518	0.8626	0.0108	8.5706	12.1812	3.6106	121.70
0.4000	0.7987	0.8008	0.0021	9.2546	13.2674	4.0128	139.75
0.4500	0.7350	0.7226	-.0124	9.8536	14.0378	4.1842	158.53
0.5000	0.6625	0.6362	-.0263	10.3160	15.1458	4.8298	170.38
0.5500	0.5844	0.5424	-.0420	10.6029	15.4672	4.8643	174.24
0.6000	0.5050	0.4476	-.0574	10.6981	15.3064	4.6083	157.69
0.6500	0.4282	0.3606	-.0676	10.6099	14.7992	4.1893	131.73
0.7000	0.3572	0.2854	-.0718	10.3666	14.1424	3.7758	111.18
0.7500	0.2942	0.2204	-.0738	10.0067	13.5246	3.5179	92.26
0.8000	0.2398	0.1728	-.0670	9.5701	12.6624	3.0923	75.17
0.9000	0.1561	0.0974	-.0587	8.5991	11.0484	2.4493	48.64
1.0000	0.1000	0.0514	-.0486	7.6412	9.6102	1.9690	31.28
1.2000	0.0403	0.0162	-.0241	6.0372	7.3124	1.2752	13.35

NORMAL SEQUENTIAL SAMPLING

TEST: NULL HYPOTHESIS: THETA = 0.0

TEST: ALTERNATE HYPOTHESIS: THETA = 1.000

TYPE I ERROR = 0.010 PROB(REJECT NULL HYP. GIVEN NULL HYP. TRUE)

TYPE II ERROR = 0.200 PROB(ACCEPT NULL HYP. GIVEN ALT. HYP. TRUE)

STANDARD DEVIATION = 1.0

THETA	OPERATING CHARACTERISTIC VALUE			EXPECTED SAMPLE SIZE			VARIANCE S S
	APPROX	EXACT	DIFFERENCE	APPROX.	EXACT	DIFFERENCE	EXACT
-.2000	0.9981	0.9994	0.0013	2.2683	3.3506	1.0823	5.97
0.0	0.9900	0.9950	0.0050	3.0792	4.5148	1.4356	14.55
0.2000	0.9542	0.9642	0.0100	4.4187	6.6540	2.2353	41.74
0.3000	0.9099	0.9160	0.0061	5.3015	8.1488	2.8473	63.43
0.4000	0.8367	0.8328	-.0039	6.2238	9.7750	3.5512	92.77
0.5000	0.7326	0.6978	-.0348	7.0086	11.0444	4.0358	109.46
0.5500	0.6717	0.6170	-.0547	7.2899	11.5676	4.2777	109.38
0.6000	0.6076	0.5364	-.0712	7.4764	11.6048	4.1284	100.48
0.6500	0.5429	0.4520	-.0909	7.5640	11.7344	4.1704	92.68
0.7000	0.4799	0.3816	-.0983	7.5582	11.4304	3.8722	81.88
0.7500	0.4203	0.3242	-.0961	7.4713	10.8988	3.4275	65.61
0.8000	0.3655	0.2634	-.1021	7.3194	10.5522	3.2328	56.64
0.8500	0.3160	0.2142	-.1018	7.1192	10.0970	2.9778	47.55
0.9000	0.2721	0.1694	-.1027	6.8863	9.6328	2.7465	38.81
1.0000	0.2000	0.1084	-.0916	6.3715	8.6188	2.2473	26.33
1.2000	0.1063	0.0460	-.0603	5.3514	6.8604	1.5090	12.06
1.4000	0.0562	0.0184	-.0378	4.4956	5.6808	1.1852	6.26

NORMAL SEQUENTIAL SAMPLING

TEST: NULL HYPOTHESIS: THETA = 0.0

TEST: ALTERNATE HYPOTHESIS: THETA = 1.000

TYPE I ERROR = 0.100 PROB(REJECT NULL HYP. GIVEN NULL HYP. TRUE)

TYPE II ERROR = 0.001 PROB(ACCEPT NULL HYP. GIVEN ALT. HYP. TRUE)

STANDARD DEVIATION = 1.0

THETA	OPERATING CHARACTERISTIC VALUE			EXPECTED SAMPLE SIZE			VARIANCE S S
	APPROX.	EXACT	DIFFERENCE	APPROX.	EXACT	DIFFERENCE	EXACT
-.2000	0.9601	0.9836	0.0235	9.1993	10.5670	1.3677	20.61
0.0	0.9000	0.9482	0.0482	11.7840	14.0220	2.2380	51.05
0.1000	0.8420	0.9042	0.0622	13.4091	16.2416	2.8325	85.58
0.2000	0.7519	0.8256	0.0737	15.1442	18.9540	3.8098	150.68
0.2500	0.6909	0.7660	0.0751	15.9535	20.3492	4.3957	209.33
0.3000	0.6179	0.6968	0.0789	16.6202	21.7110	5.0908	272.57
0.3500	0.5334	0.6018	0.0684	17.0304	22.3676	5.3372	334.14
0.4000	0.4402	0.5036	0.0634	17.0581	23.0580	5.9999	415.32
0.4500	0.3440	0.3908	0.0468	16.6054	22.8426	6.2372	451.95
0.5000	0.2528	0.2786	0.0258	15.6563	21.4654	5.8091	451.73
0.5500	0.1742	0.1822	0.0080	14.3063	19.6530	5.3467	409.62
0.6000	0.1129	0.1108	-.0021	12.7353	17.5104	4.7751	332.23
0.7000	0.0407	0.0366	-.0041	9.6568	12.7682	3.1114	162.96
0.8000	0.0127	0.0094	-.0033	7.2867	9.5352	2.2485	80.98
1.0000	0.0010	0.0004	-.0006	4.5850	6.0190	1.4340	21.90
1.2000	0.0001	0.0	-.0001	3.2871	4.4062	1.1191	8.28

NORMAL SEQUENTIAL SAMPLING

TEST: NULL HYPOTHESIS: THETA = 0.0

TEST: ALTERNATE HYPOTHESIS: THETA = 1.000

TYPE I ERROR = 0.100 PROB(REJECT NULL HYP. GIVEN NULL HYP. TRUE)

TYPE II ERROR = 0.010 PROB(ACCEPT NULL HYP. GIVEN ALT. HYP. TRUE)

STANDARD DEVIATION = 1.0

THETA	OPERATING CHARACTERISTIC VALUE			EXPECTED SAMPLE SIZE			VARIANCE S S
	APPROX.	EXACT	DIFFERENCE	APPROX.	EXACT	DIFFERENCE	EXACT
-.2000	0.9597	0.9830	0.0233	6.0372	7.3642	1.3270	13.46
0.0	0.9000	0.9480	0.0480	7.6412	9.5824	1.9412	30.91
0.1500	0.8060	0.8682	0.0622	9.0918	11.8750	2.7832	60.53
0.2000	0.7602	0.8308	0.0706	9.5701	12.6894	3.1193	79.41
0.2500	0.7058	0.7812	0.0754	10.0067	13.6744	3.6677	94.86
0.3000	0.6428	0.7156	0.0728	10.3666	14.4224	4.0558	112.12
0.3500	0.5718	0.6374	0.0656	10.6099	15.0620	4.4521	134.70
0.4000	0.4950	0.5478	0.0528	10.6981	15.2014	4.5033	148.45
0.4500	0.4156	0.4538	0.0382	10.6030	15.1744	4.5714	166.93
0.5000	0.3375	0.3580	0.0205	10.3160	14.9740	4.6580	166.95
0.5500	0.2650	0.2706	0.0056	9.8536	14.1474	4.2938	157.43
0.6000	0.2013	0.1974	-.0039	9.2546	13.2164	3.9618	143.92
0.6500	0.1482	0.1366	-.0116	8.5705	12.0772	3.5067	116.73
0.7000	0.1063	0.0946	-.0117	7.8540	10.9330	3.0790	95.90
0.8000	0.0511	0.0392	-.0119	6.4849	8.7946	2.3097	56.20
1.0000	0.0100	0.0052	-.0048	4.4492	5.9148	1.4656	19.66
1.2000	0.0018	0.0010	-.0008	3.2580	4.3772	1.1192	8.12

NORMAL SEQUENTIAL SAMPLING

TEST: NULL HYPOTHESIS: THETA = 0.0

TEST: ALTERNATE HYPOTHESIS: THETA = 1.000

TYPE I ERROR = 0.100 PROB(REJECT NULL HYP. GIVEN NULL HYP. TRUE)

TYPE II ERROR = 0.100 PROB(ACCEPT NULL HYP. GIVEN ALT. HYP. TRUE)

STANDARD DEVIATION = 1.0

THETA	OPERATING CHARACTERISTIC VALUE			EXPECTED SAMPLE SIZE			VARIANCE S S
	APPROX.	EXACT	DIFFERENCE	APPROX.	EXACT	DIFFERENCE	EXACT
-.2000	0.9559	0.9814	0.0255	2.8620	4.0600	1.1980	6.50
0.0	0.9000	0.9448	0.0448	3.5156	5.1878	1.6722	13.54
0.1000	0.8529	0.9076	0.0547	3.8774	5.9184	2.0410	18.79
0.2000	0.7889	0.8432	0.0543	4.2319	6.6454	2.4135	25.99
0.3000	0.7066	0.7506	0.0440	4.5393	7.3406	2.8013	32.45
0.3500	0.6591	0.6972	0.0381	4.6603	7.5976	2.9373	35.65
0.4000	0.6081	0.6330	0.0249	4.7516	7.7718	3.0202	37.61
0.4500	0.5547	0.5630	0.0083	4.8085	7.8618	3.0533	38.15
0.5000	0.5000	0.4948	-.0052	4.8278	7.8872	3.0594	39.68
0.5500	0.4453	0.4272	-.0181	4.8085	7.8620	3.0535	38.06
0.6000	0.3919	0.3596	-.0323	4.7516	7.7374	2.9858	37.83
0.6500	0.3409	0.2992	-.0417	4.6603	7.5330	2.8727	34.68
0.7000	0.2934	0.2462	-.0472	4.5393	7.1666	2.6273	30.19
0.8000	0.2111	0.1616	-.0495	4.2319	6.5122	2.2803	22.76
0.9000	0.1471	0.0988	-.0483	3.8773	5.8626	1.9853	17.95
1.0000	0.1000	0.0612	-.0388	3.5156	5.1452	1.6296	13.04
1.2000	0.0441	0.0234	-.0207	2.8620	4.0826	1.2206	6.39

NORMAL SEQUENTIAL SAMPLING

TEST: NULL HYPOTHESIS: THETA = 0.0

TEST: ALTERNATE HYPOTHESIS: THETA = 1.000

TYPE I ERROR = 0.100 PROB(REJECT NULL HYP. GIVEN NULL HYP. TRUE)

TYPE II ERROR = 0.200 PROB(ACCEPT NULL HYP. GIVEN ALT. HYP. TRUE)

STANDARD DEVIATION = 1.0

THETA	OPERATING CHARACTERISTIC VALUE			EXPECTED SAMPLE SIZE			VARIANCE S S
	APPROX	EXACT	DIFFERENCE	APPROX.	EXACT	DIFFERENCE	EXACT
-.2000	0.9519	0.9780	0.0261	1.9024	3.0932	1.1908	4.60
0.0	0.9000	0.9384	0.0384	2.2915	3.8036	1.5121	7.80
0.1000	0.8594	0.8994	0.0400	2.5008	4.2438	1.7430	10.78
0.2000	0.8068	0.8476	0.0408	2.7057	4.7246	2.0189	14.06
0.3000	0.7416	0.7716	0.0300	2.8903	5.1980	2.3077	17.40
0.4000	0.6650	0.6754	0.0104	3.0363	5.6062	2.5699	21.13
0.5000	0.5803	0.5622	-.0181	3.1276	5.7652	2.6376	22.23
0.6000	0.4922	0.4448	-.0474	3.1545	5.7636	2.6091	20.47
0.7000	0.4063	0.3370	-.0693	3.1168	5.5426	2.4258	17.57
0.8000	0.3272	0.2470	-.0802	3.0229	5.2302	2.2073	14.62
0.9000	0.2580	0.1668	-.0912	2.8872	4.9138	2.0266	12.20
1.0000	0.2000	0.1180	-.0820	2.7255	4.4550	1.7295	9.20
1.2000	0.1159	0.0534	-.0625	2.3773	3.7302	1.3529	5.29
1.3000	0.0872	0.0374	-.0498	2.2088	3.4122	1.2034	4.18

NORMAL SEQUENTIAL SAMPLING

TEST: NULL HYPOTHESIS: THETA = 0.0

TEST: ALTERNATE HYPOTHESIS: THETA = 2.000

TYPE I ERROR = 0.010 PROB(REJECT NULL HYP. GIVEN NULL HYP. TRUE)

TYPE II ERROR = 0.010 PROB(ACCEPT NULL HYP. GIVEN ALT. HYP. TRUE)

STANDARD DEVIATION = 1.0

THETA	OPERATING CHARACTERISTIC VALUE			EXPECTED SAMPLE SIZE			VARIANCE S S
	APPROX.	EXACT	DIFFERENCE	APPROX.	EXACT	DIFFERENCE	EXACT
-.4000	0.9984	0.9996	0.0012	1.6358	2.3424	0.7066	1.15
0.0	0.9900	0.9984	0.0084	2.2516	3.1408	0.8892	2.97
0.2500	0.9691	0.9880	0.0189	2.8742	4.0002	1.1260	5.80
0.5000	0.9087	0.9494	0.0407	3.7558	5.4510	1.6952	14.31
0.6000	0.8627	0.9128	0.0501	4.1668	6.2348	2.0680	20.98
0.7000	0.7988	0.8514	0.0526	4.5761	7.0706	2.4945	29.81
0.8000	0.7148	0.7586	0.0438	4.9362	7.7778	2.8416	36.61
0.9000	0.6129	0.6406	0.0277	5.1878	8.3364	3.1486	43.56
1.0000	0.5000	0.4970	-.0030	5.2788	8.6196	3.3408	45.35
1.1000	0.3871	0.3610	-.0261	5.1878	8.2314	3.0436	43.15
1.2000	0.2852	0.2374	-.0478	4.9362	7.6850	2.7488	35.89
1.3000	0.2012	0.1514	-.0498	4.5761	6.8832	2.3071	25.45
1.4000	0.1373	0.0920	-.0453	4.1669	6.1246	1.9577	19.44
1.5000	0.0913	0.0544	-.0369	3.7558	5.4036	1.6478	14.31
1.7500	0.0309	0.0162	-.0147	2.8743	3.9966	1.1223	5.77
2.0000	0.0100	0.0038	-.0062	2.2516	3.1774	0.9258	2.98
2.4000	0.0016	0.0	-.0016	1.6359	2.3642	0.7283	1.20

NORMAL SEQUENTIAL SAMPLING

TEST: NULL HYPOTHESIS: THETA = 0.0

TEST: ALTERNATE HYPOTHESIS: THETA = 2.000

TYPE I ERROR = 0.010 PROB(REJECT NULL HYP. GIVEN NULL HYP. TRUE)

TYPE II ERROR = 0.100 PROB(ACCEPT NULL HYP. GIVEN ALT. HYP. TRUE)

STANDARD DEVIATION = 1.0

THETA	OPERATING CHARACTERISTIC VALUE			EXPECTED SAMPLE SIZE			VARIANCE S S
	APPROX	EXACT	DIFFERENCE	APPROX.	EXACT	DIFFERENCE	EXACT
-.4000	0.9982	0.9994	0.0012	0.8145	1.5332	0.7187	0.64
0.0	0.9900	0.9982	0.0082	1.1123	1.9826	0.8703	1.65
0.2500	0.9717	0.9904	0.0187	1.4004	2.5012	1.1008	3.44
0.5000	0.9256	0.9500	0.0244	1.7872	3.2238	1.4366	6.57
0.7000	0.8518	0.8760	0.0242	2.1426	4.0882	1.9456	11.87
0.8000	0.7987	0.8098	0.0111	2.3136	4.4898	2.1762	15.06
0.9000	0.7350	0.7228	-.0122	2.4634	4.9338	2.4704	18.24
1.0000	0.6625	0.6206	-.0419	2.5790	5.1838	2.6048	18.81
1.1000	0.5844	0.5114	-.0730	2.6507	5.1946	2.5439	17.73
1.2000	0.5050	0.4064	-.0986	2.6745	5.1878	2.5133	16.17
1.3000	0.4282	0.3160	-.1122	2.6525	4.9178	2.2653	12.76
1.4000	0.3572	0.2326	-.1246	2.5917	4.7066	2.1149	11.00
1.5000	0.2942	0.1674	-.1268	2.5017	4.4040	1.9023	8.90
1.6000	0.2398	0.1244	-.1154	2.3925	4.0550	1.6625	6.53
1.7000	0.1940	0.0934	-.1006	2.2730	3.7220	1.4490	5.07
1.8000	0.1561	0.0650	-.0911	2.1498	3.4594	1.3096	4.05
2.0000	0.1000	0.0288	-.0712	1.9103	3.0058	1.0955	2.62
2.4000	0.0403	0.0074	-.0329	1.5093	2.3030	0.7937	1.14

NORMAL SEQUENTIAL SAMPLING

TEST: NULL HYPOTHESIS: THETA = 0.0

TEST: ALTERNATE HYPOTHESIS: THETA = 2.000

TYPE I ERROR = 0.100 PROB(REJECT NULL HYP. GIVEN NULL HYP. TRUE)

TYPE II ERROR = 0.010 PROB(ACCEPT NULL HYP. GIVEN ALT. HYP. TRUE)

STANDARD DEVIATION = 1.0

THETA	OPERATING CHARACTERISTIC VALUE			EXPECTED SAMPLE SIZE			VARIANCE S S
	APPROX.	EXACT	DIFFERENCE	APPROX.	EXACT	DIFFERENCE	EXACT
-.4000	0.9597	0.9950	0.0353	1.5093	2.2948	0.7855	1.08
0.0	0.9000	0.9728	0.0728	1.9103	2.9858	1.0755	2.51
0.2000	0.8439	0.9368	0.0929	2.1498	3.4346	1.2848	3.82
0.3000	0.8060	0.9134	0.1074	2.2730	3.7184	1.4454	5.12
0.4000	0.7602	0.8806	0.1204	2.3925	4.0564	1.6639	6.65
0.5000	0.7058	0.8304	0.1246	2.5017	4.3642	1.8625	8.31
0.6000	0.6428	0.7648	0.1220	2.5917	4.6530	2.0613	10.68
0.7000	0.5718	0.6922	0.1204	2.6525	5.0078	2.3553	14.23
0.8000	0.4950	0.5930	0.0980	2.6745	5.1498	2.4753	15.77
0.9000	0.4156	0.4882	0.0726	2.6507	5.2096	2.5589	17.83
1.0000	0.3375	0.3794	0.0419	2.5790	5.1754	2.5964	18.09
1.1000	0.2650	0.2750	0.0100	2.4634	4.8986	2.4352	17.34
1.2000	0.2013	0.1924	-.0089	2.3136	4.5506	2.2370	14.59
1.3500	0.1259	0.1044	-.0215	2.0533	3.8654	1.8121	10.36
1.5000	0.0744	0.0506	-.0238	1.7872	3.3310	1.5438	7.54
1.7500	0.0283	0.0140	-.0143	1.4004	2.5232	1.1228	3.42
2.0000	0.0100	0.0038	-.0062	1.1123	2.0372	0.9249	1.76
2.4000	0.0018	0.0	-.0018	0.8145	1.5534	0.7389	0.68

NORMAL SEQUENTIAL SAMPLING

TEST: NULL HYPOTHESIS: THETA = 0.0

TEST: ALTERNATE HYPOTHESIS: THETA = 2.000

TYPE I ERROR = 0.100 PROB(REJECT NULL HYP, GIVEN NULL HYP, TRUE)

TYPE II ERROR = 0.100 PROB(ACCEPT NULL HYP. GIVEN ALT, HYP. TRUE)

STANDARD DEVIATION = 1.0

THETA	OPERATING CHARACTERISTIC VALUE			EXPECTED SAMPLE SIZE			VARIANCE S S
	APPROX	EXACT	DIFFERENCE	APPROX.	EXACT	DIFFERENCE	EXACT
-.4000	0.9559	0.9930	0.0371	0.7155	1.4870	0.7715	0.58
0.0	0.9000	0.9710	0.0710	0.8789	1.8528	0.9739	1.25
0.2000	0.8529	0.9380	0.0851	0.9693	2.1040	1.1347	1.78
0.4000	0.7889	0.8844	0.0955	1.0580	2.4022	1.3442	2.77
0.6000	0.7066	0.7976	0.0910	1.1348	2.7266	1.5918	3.83
0.7000	0.6591	0.7386	0.0795	1.1651	2.8858	1.7207	4.44
0.8000	0.6081	0.6670	0.0589	1.1879	2.9668	1.7789	5.02
0.9000	0.5547	0.5874	0.0327	1.2021	3.0080	1.8059	5.22
1.0000	0.5000	0.5088	0.0088	1.2069	3.0790	1.8721	5.62
1.1000	0.4453	0.4234	-.0219	1.2021	3.0432	1.8411	5.44
1.2000	0.3919	0.3462	-.0457	1.1879	2.9454	1.7575	5.11
1.3000	0.3409	0.2758	-.0651	1.1651	2.8466	1.6815	4.66
1.4000	0.2934	0.2120	-.0814	1.1348	2.7230	1.5832	3.97
1.5000	0.2500	0.1632	-.0868	1.0986	2.5764	1.4778	3.33
1.7500	0.1614	0.0806	-.0808	0.9920	2.1956	1.2036	2.11
2.0000	0.1000	0.0330	-.0670	0.8789	1.8916	1.0127	1.29
2.4000	0.0441	0.0094	-.0347	0.7155	1.5080	0.7925	0.53

NORMAL SEQUENTIAL SAMPLING

TEST: NULL HYPOTHESIS: THETA = 0.0

TEST: ALTERNATE HYPOTHESIS: THETA = 3.000

TYPE I ERROR = 0.010 PROB(REJECT NULL HYP. GIVEN NULL HYP. TRUE)

TYPE II ERROR = 0.010 PROB(ACCEPT NULL HYP. GIVEN ALT. HYP. TRUE)

STANDARD DEVIATION = 1.0

THETA	OPERATING CHARACTERISTIC VALUE			EXPECTED SAMPLE SIZE			VARIANCE S S
	APPROX.	EXACT	DIFFERENCE	APPROX.	EXACT	DIFFERENCE	EXACT
-.3000	0.9960	0.9996	0.0036	0.8441	1.4560	0.6119	0.42
0.0	0.9900	0.9988	0.0088	1.0007	1.6748	0.6741	0.68
0.6000	0.9403	0.9794	0.0391	1.4987	2.5120	1.0133	2.48
0.9000	0.8627	0.9288	0.0661	1.8519	3.2372	1.3853	4.81
1.0500	0.7988	0.8692	0.0704	2.0338	3.7082	1.6744	7.10
1.2000	0.7148	0.7830	0.0682	2.1938	4.1412	1.9474	9.98
1.2750	0.6658	0.7206	0.0548	2.2574	4.3284	2.0710	11.02
1.3500	0.6129	0.6558	0.0429	2.3057	4.5590	2.2533	12.73
1.4250	0.5572	0.5790	0.0218	2.3359	4.5966	2.2607	13.08
1.5000	0.5000	0.5002	0.0002	2.3461	4.6342	2.2881	13.40
1.5750	0.4428	0.4234	-.0194	2.3358	4.6000	2.2642	12.98
1.6500	0.3871	0.3476	-.0395	2.3057	4.4904	2.1847	12.66
1.7250	0.3342	0.2824	-.0518	2.2574	4.3588	2.1014	10.89
1.8000	0.2852	0.2200	-.0652	2.1938	4.2246	2.0308	10.50
1.9500	0.2012	0.1336	-.0676	2.0338	3.7258	1.6920	7.48
2.1000	0.1373	0.0798	-.0575	1.8519	3.2534	1.4015	5.30
2.4000	0.0597	0.0186	-.0411	1.4987	2.5444	1.0457	2.50
2.7000	0.0247	0.0066	-.0181	1.2134	2.0478	0.8344	1.29
3.0000	0.0100	0.0012	-.0088	1.0007	1.7078	0.7071	0.73
3.3000	0.0040	0.0006	-.0034	0.8441	1.4712	0.6271	0.44

NORMAL SEQUENTIAL SAMPLING

TEST: NULL HYPOTHESIS: THETA = 0.0

TEST: ALTERNATE HYPOTHESIS: THETA = 3.000

TYPE I ERROR = 0.010 PROB(REJECT NULL HYP. GIVEN NULL HYP. TRUE)

TYPE II ERROR = 0.100 PROB(ACCEPT NULL HYP. GIVEN ALT. HYP. TRUE)

STANDARD DEVIATION = 1.0

THETA	OPERATING CHARACTERISTIC VALUE			EXPECTED SAMPLE SIZE			VARIANCE S S
	APPROX.	EXACT	DIFFERENCE	APPROX.	EXACT	DIFFERENCE	EXACT
-.3000	0.9958	0.9996	0.0038	0.4192	1.1654	0.7462	0.18
0.0	0.9900	0.9988	0.0088	0.4944	1.2814	0.7870	0.34
0.6000	0.9489	0.9804	0.0315	0.7205	1.7334	1.0129	1.28
0.9000	0.8937	0.9324	0.0387	0.8727	2.1522	1.2795	2.50
1.0500	0.8518	0.8874	0.0356	0.9523	2.4060	1.4537	3.32
1.2000	0.7987	0.8184	0.0197	1.0283	2.6858	1.6575	4.39
1.3500	0.7350	0.7250	-.0100	1.0949	2.9612	1.8663	5.57
1.5000	0.6625	0.6076	-.0549	1.1462	3.0530	1.9068	5.98
1.6500	0.5844	0.4874	-.0970	1.1781	3.0620	1.8839	5.56
1.8000	0.5050	0.3690	-.1360	1.1887	3.0344	1.8457	5.20
1.9500	0.4282	0.2646	-.1636	1.1789	2.8834	1.7045	4.01
2.1000	0.3572	0.1902	-.1670	1.1518	2.6626	1.5108	3.08
2.2500	0.2942	0.1244	-.1698	1.1119	2.4998	1.3879	2.52
2.4000	0.2398	0.0876	-.1522	1.0633	2.2754	1.2121	1.84
2.7000	0.1561	0.0384	-.1177	0.9555	1.9226	0.9671	1.06
2.8500	0.1251	0.0210	-.1041	0.9013	1.7846	0.8833	0.83
3.0000	0.1000	0.0140	-.0860	0.8490	1.6622	0.8132	0.65
3.3000	0.0636	0.0054	-.0582	0.7533	1.4532	0.6999	0.42

NORMAL SEQUENTIAL SAMPLING

TEST: NULL HYPOTHESIS: THETA = 0.0

TEST: ALTERNATE HYPOTHESIS: THETA = 3.000

TYPE I ERROR = 0.100 PROB(REJECT NULL HYP. GIVEN NULL HYP. TRUE)

TYPE II ERROR = 0.010 PROB(ACCEPT NULL HYP. GIVEN ALT. HYP. TRUE)

STANDARD DEVIATION = 1.0

THETA	OPERATING CHARACTERISTIC VALUE			EXPECTED SAMPLE SIZE			VARIANCE S S
	APPROX.	EXACT	DIFFERENCE	APPROX.	EXACT	DIFFERENCE	EXACT
-.6000	0.9597	0.9982	0.0385	0.6708	1.2912	0.6204	0.26
0.0	0.9000	0.9886	0.0886	0.8490	1.6348	0.7858	0.62
0.3000	0.8439	0.9696	0.1257	0.9555	1.9086	0.9531	1.04
0.4500	0.8060	0.9514	0.1454	1.0102	2.0756	1.0654	1.35
0.6000	0.7602	0.9178	0.1576	1.0633	2.2404	1.1771	1.73
0.7500	0.7058	0.8812	0.1754	1.1119	2.4804	1.3685	2.40
0.9000	0.6428	0.8250	0.1822	1.1518	2.6888	1.5370	3.05
1.0500	0.5718	0.7462	0.1744	1.1789	2.8966	1.7177	4.03
1.2000	0.4950	0.6482	0.1532	1.1887	3.0366	1.8479	4.89
1.3500	0.4156	0.5302	0.1146	1.1781	3.1080	1.9299	5.83
1.5000	0.3375	0.4066	0.0691	1.1462	3.0606	1.9144	5.66
1.6500	0.2650	0.2870	0.0220	1.0948	2.9638	1.8690	5.62
1.8000	0.2013	0.1904	-.0109	1.0283	2.7316	1.7033	4.93
1.9500	0.1482	0.1210	-.0272	0.9523	2.4500	1.4977	3.48
2.2500	0.0744	0.0396	-.0348	0.7943	1.9774	1.1831	1.95
2.7000	0.0231	0.0070	-.0161	0.5933	1.4842	0.8909	0.69
3.0000	0.0100	0.0016	-.0084	0.4944	1.2956	0.8012	0.37
3.6000	0.0018	0.0	-.0018	0.3620	1.1038	0.7418	0.11

NORMAL SEQUENTIAL SAMPLING

TEST: NULL HYPOTHESIS: THETA = 0.0

TEST: ALTERNATE HYPOTHESIS: THETA = 3.000

TYPE I ERROR = 0.100 PROB(REJECT NULL HYP. GIVEN NULL HYP. TRUE)

TYPE II ERROR = 0.100 PROB(ACCEPT NULL HYP. GIVEN ALT. HYP. TRUE)

STANDARD DEVIATION = 1.0

THETA	OPERATING CHARACTERISTIC VALUE			EXPECTED SAMPLE SIZE			VARIANCE S S
	APPROX.	EXACT	DIFFERENCE	APPROX.	EXACT	DIFFERENCE	EXACT
-.6000	0.9559	0.9978	0.0419	0.3180	1.0838	0.7658	0.09
0.0	0.9000	0.9880	0.0880	0.3906	1.2486	0.8580	0.29
0.3000	0.8529	0.9696	0.1167	0.4308	1.3834	0.9526	0.49
0.4500	0.8232	0.9510	0.1278	0.4509	1.4674	1.0165	0.62
0.7500	0.7500	0.8912	0.1412	0.4883	1.6698	1.1815	1.01
0.9750	0.6833	0.8108	0.1275	0.5115	1.8386	1.3271	1.39
1.2000	0.6081	0.6984	0.0903	0.5280	1.9702	1.4422	1.77
1.3500	0.5547	0.6054	0.0507	0.5343	2.0578	1.5235	2.07
1.5000	0.5000	0.5084	0.0084	0.5364	2.0774	1.5410	2.12
1.6500	0.4453	0.4098	-.0355	0.5343	2.0548	1.5205	2.10
1.8000	0.3919	0.3182	-.0737	0.5280	2.0012	1.4732	1.99
2.1000	0.2934	0.1710	-.1224	0.5044	1.7860	1.2816	1.23
2.4000	0.2111	0.0834	-.1277	0.4702	1.5758	1.1056	0.83
2.7000	0.1471	0.0378	-.1093	0.4308	1.3952	0.9644	0.50
3.0000	0.1000	0.0146	-.0854	0.3906	1.2562	0.8656	0.30
3.6000	0.0441	0.0024	-.0417	0.3180	1.0930	0.7750	0.1[illegible]

COMPUTER PROGRAM -- APPROXIMATE BINOMIAL

```
C INPUT VARIABLES ARE:
C
C          1. ICASE   - # OF CASES TO SIMULATE
C          2. THETA0  - NULL HYPOTHESIS (H) VARIABLE
C          3. THETA1  - ALTERNATE HYPOTHESIS (A) VARIABLE
C          4. ALFA    - TYPE I  ERROR
C          5. BETA    - TYPE II ERROR
C
      IMPLICIT REAL (A-H, N-Z)
      READ(5, 1) ICASE
    1 FORMAT(I10)
      DO 7  K = 1, ICASE
      READ(5, 2)THETA0, THETA1, ALFA, BETA
    2 FORMAT(4F10.0)
C
C COMPUTE APPROXIMATE A & B
      A = (1-BETA)/ALFA
      B = BETA/(1-ALFA)
C
      WRITE(6, 3)THETA0,THETA1,ALFA,BETA,A,B
    3 FORMAT(1H1,9X,'TEST: NULL HYPOTHESIS: THETA = ',F7.3,/
     */10X,'TEST: ALTERNATE HYPOTHESIS: THETA = ',F7.3,//
     *10X,'TYPE  I ERROR = ',F5.3,'     PROB(REJECT NULL HYP'
     *,'. GIVEN NULL HYP. TRUE)' ,//10X,'TYPE II ERROR = ',
     *F5.3,'     PROB(ACCEPT NULL HYP. GIVEN ALT. HYP. TRUE)'
     *,//10X,'ACCEPTANCE BOUNDRY, A = ',F9.4, //10X,
     *'REJECTION BOUNDRY,  B = ',F9.5,//10X,'AVERAGE SAMPLE'
     *,' NUMBER:',T39,'THETA:',T51,'OPERATING CHARACTERISTI'
     *,'C VALUE:',//T18,'ORDINATE',T38,'ABSCISSA',T63,
     *'ORDINATE',//)
C
      AA = ALOG(10000*(1-BETA)) - ALOG(10000*ALFA)
      BB = ALOG(10000*(1-ALFA)) - ALOG(10000*BETA)
      C = 1.0/BB
      D = 0.0
      E = 1.0
      F = 1-ALFA
      G1 = ALOG(10000*THETA1) - ALOG(10000*THETA0)
      G2 = ALOG(10000*(1-THETA0)) - ALOG(10000*(1-THETA1))
      H1 = BB/(G1+G2)
      H2 = AA/(G1+G2)
      NBS0 = BB/G2
      WRITE(6, 4)NBS0,D,E
    4 FORMAT(T16,F9.4,T37,F9.6,T62,F9.6)
      WRITE(7, 5)NBS0,D,E
    5 FORMAT(F10.4,F10.6,F10.6)
      NBST0 = (((1-ALFA)*BB) - (ALFA*AA))/(G2-(THETA0*(G1+G2
     *)))
      WRITE(6, 4)NBST0,THETA0,F
      WRITE(7, 5)NBST0,THETA0,F
      NBSS = (AA*BB)/(G1*G2)
      OSS = H2/(H1+H2)
      S = G2/(G1+G2)
      WRITE(6, 4)NBSS,S,OSS
      WRITE(7, 5)NBSS,S,OSS
      NBST1 = (((1-BETA)*AA) - (BETA*BB))/((THETA1*(G1+G2))-
     *G2)
      WRITE(6, 4)NBST1,THETA1,BETA
      WRITE(7, 5)NBST1,THETA1,BETA
      NBS1 = AA/G1
      WRITE(6, 4)NBS1,E,D
      WRITE(7, 5)NBS1,E,D
      Q1 = EXP(.1*(G1+G2))
      Q2 = EXP(S * .1 * (G1+G2))
      Q3 = EXP(H1 * .1 * (G1+G2))
      Q4 = EXP((H1+H2)*.1*(G1+G2))
      I = 1
```

```
6 R1 = Q1 ** I
  R2 = Q2 ** I
  R3 = Q3 ** I
  R4 = Q4 ** I
  PPRIME = (R2-1)/(R1-1)
  QSUBPP = (R4-R3)/(R4-1)
  NBSPP = (((H1+H2)*QSUBPP)-H2)/(S-PPRIME)
  WRITE(6, 4)NBSPP,PPRIME,QSUBPP
  WRITE(7, 5)NBSPP,PPRIME,QSUBPP
  PDPRIM = (PPRIME * R1)/R2
  QSUBDP = QSUBPP/R3
  NBSPDP = ((H2-((H1+H2)*QSUBDP)))/(PDPRIM-S)
  WRITE(6, 4)NBSPDP,PDPRIM,QSUBDP
  WRITE(7, 5)NBSPDP,PDPRIM,QSUBDP
  I = I + 2
C
  IF(I.LT.41)GO TO 6
C
7 CONTINUE
  CALL EXIT
  END
```

COMPUTER PROGRAM -- APPROXIMATE NORMAL

```
C INPUT VARIABLES ARE:
C
C         1. ICASE   - # OF CASES TO SIMULATE
C         2. THETA0  - NULL HYPOTHESIS (H) VARIABLE
C         3. THETA1  - ALTERNATE HYPOTHESIS (A) VARIABLE
C         4. ALFA    - TYPE I  ERROR
C         5. BETA    - TYPE II ERROR
C         6. STDX    -  STANDARD DEVIATION OF SAMPLE DIST'N.
C
      IMPLICIT REAL (A-H, N-Z)
      READ(5, 1) ICASE
    1 FORMAT(I10)
      DO 8 K = 1,ICASE
      READ(5, 2)THETA0, THETA1, ALFA, BETA, STDX
    2 FORMAT(5F10.0)
C
C COMPUTE APPROXIMATE A & B
C
      A = (1-BETA)/ALFA
      B = BETA/(1-ALFA)
C
      WRITE(6, 3)THETA0, THETA1, ALFA, BETA, STDX, A, B
    3 FORMAT(1H1,9X,'TEST: NULL HYPOTHESIS: THETA = ',F7.3,/
     */10X,'TEST: ALTERNATE HYPOTHESIS: THETA = ',F7.3,//
     *10X,'TYPE  I ERROR = ',F5.3,'     PROB(REJECT NULL HYP'
     *,'. GIVEN NULL HYP. TRUE)' ,//10X,'TYPE II ERROR = ',
     *F5.3,'     PROB(ACCEPT NULL HYP. GIVEN ALT. HYP. TRUE)'
     *,//10X,'STANDARD DEVIATION = ', F5.1
     *,//10X,'ACCEPTANCE BOUNDRY, A = ',F9.4, //10X,
     *'REJECTION BOUNDRY,  B = ',F9.5,//10X,'AVERAGE SAMPLE'
     *,' NUMBER:',T39,'THETA:',T51,'OPERATING CHARACTERISTI'
     *,'C VALUE:',//T18,'ORDINATE',T38,'ABSCISSA',T63,
     *'ORDINATE',//)
C
      AA = ALOG(10000*(1-BETA)) - ALOG(10000*ALFA)
      BB = ALOG(10000*(1-ALFA)) - ALOG(10000*BETA)
C
      XM2 = THETA1 + (.4 * THETA1)
      XM = THETA0 - (.5 * THETA1)
C
    4 XM = XM + (.025 * THETA1)
      XH1 = BB * (STDX**2)/(THETA1 - THETA0)
      XH2 = AA * (STDX**2)/(THETA1 - THETA0)
      S = (THETA0 + THETA1)/2.0
      IF(ABS(XM-S).LE..0001)GO TO 7
      XK = (2 * (S - XM))/(STDX ** 2)
      T1 = XH1 * XK
      T2 = (XH1 + XH2) * XK
      OSTC = (EXP(T1) - 1)/(EXP(T2) - 1)
      OST = 1.0 - OSTC
      NBST = (OSTC*(XH1 + XH2) - XH1)/(XM - S)
      WRITE(6,5) NBST,XM,OST
    5 FORMAT(T16,F9.4,T37,F9.6,T62,F9.6)
      WRITE(7,6) NBST,XM,OST
    6 FORMAT(F10.4,F10.5,F10.6)
      IF(XM.LE.XM2)GO TO 4
      GO TO 8
    7 OSSC = XH1/(XH1+XH2)
      OSS = 1.0 - OSSC
      NBSS = (XH1 * XH2)/(STDX ** 2)
      WRITE(6, 5)NBSS,XM,OSS
      WRITE(7, 6)NBSS,XM,OSS
      GO TO 4
    8 CONTINUE
      CALL EXIT
      END
```

COMPUTER PROGRAM -- APPROXIMATE EXPONENTIAL

```
C         EXPONENTIAL DISTRIBUTION:    F(X, T) =  T * EXP(-T*X)
C
C  INPUT VARIABLES ARE:
C
C              1. ICASE   - # OF CASES TO SIMULATE
C              2. THETA0  - NULL HYPOTHESIS (H) VARIABLE         (T0)
C              3. THETA1  - ALTERNATE HYPOTHESIS (A) VARIABLE (T1)
C              4. ALFA    - TYPE I  ERROR
C              5. BETA    - TYPE II ERROR
C
      IMPLICIT REAL (A-H, L-Z)
      READ(5, 1) ICASE
    1 FORMAT(I10)
      DO 7  K = 1, ICASE
      READ(5, 2)THETA0, THETA1, ALFA, BETA
    2 FORMAT(4F10.0)
C
C  COMPUTE APPROXIMATE A & B
C
      A = (1-BETA)/ALFA
      B = BETA/(1-ALFA)
C
      WRITE(6, 3)THETA0,THETA1,ALFA,BETA,A,B
    3 FORMAT(1H1,9X,'TEST: NULL HYPOTHESIS: THETA = ',F7.3,/
     */10X,'TEST: ALTERNATE HYPOTHESIS: THETA = ',F7.3,//
     *10X,'TYPE  I ERROR = ',F5.3,'     PROB(REJECT NULL HYP'
     *,'. GIVEN NULL HYP. TRUE)' ,//10X,'TYPE II ERROR = ',
     *F5.3,'     PROB(ACCEPT NULL HYP. GIVEN ALT. HYP. TRUE)'
     *,//10X,'ACCEPTANCE BOUNDRY, A = ',F9.4, //10X,
     *'REJECTION BOUNDRY,  B = ',F9.5,//10X,'AVERAGE SAMPLE'
     *,' NUMBER:',T39,'THETA:',T51,'OPERATING CHARACTERISTI'
     *,'C VALUE:',//T18,'ORDINATE',T38,'ABSCISSA',T63,
     *'ORDINATE',//)
C
      T0 = THETA0
      T1 = THETA1
C
      DO 7 I = 1, 50
C
      L = I/10.0
      S = (T1 - T0)/ALOG(T1/T0)
      LTHETA = ((A ** L) - 1)/(A ** L - B ** L)
      T = (( L)*(T1 - T0))/(((T1/T0)**( L))-1)
      THETA = T
      EZI = ALOG(T1/T0) - (T1 - T0)/T
      IF(ABS(T-S).LE..0001) GO TO 6
      EN = ((LTHETA*ALOG(B))+(1 -LTHETA)*ALOG(A))/EZI
      WRITE(6,4)EN, THETA, LTHETA
    4 FORMAT(T16,F9.4,T37,F9.6,T62,F9.6)
      WRITE(7,5)EN, THETA, LTHETA
    5 FORMAT(F10.4,F10.6,F10.6)
      LTHETA = ((A**(-L))-1)/((A**(-L))-(B**(-L)))
      T = ((-L)*(T1 - T0))/(((T1/T0)**(-L))-1)
      THETA = T
      EZI = ALOG(T1/T0) - (T1 - T0)/T
      IF(ABS(T-S).LE..0001) GO TO 6
      EN = ((LTHETA*ALOG(B))+(1 -LTHETA)*ALOG(A))/EZI
      WRITE(6,4)EN, THETA, LTHETA
      WRITE(7,5)EN, THETA, LTHETA
      GO TO 7
    6 ENS=((-ALOG(A))*ALOG(B))/((ALOG(T1/T0)**2))
      WRITE(6,4)ENS, THETA, LTHETA
      WRITE(7,5)ENS, THETA, LTHETA
    7 CONTINUE
    8 CONTINUE
      CALL EXIT
      END
```

COMPUTER PROGRAM -- EXACT BINOMIAL

```
C INPUT VARIABLES ARE:
C
C          1. ICASE   - # OF CASES TO SIMULATE
C          2. THETAO  - NULL HYPOTHESIS (H) VARIABLE
C          3. THETA1  - ALTERNATE HYPOTHESIS (A) VARIABLE
C          4. ALFA    - TYPE I  ERROR
C          5. BETA    - TYPE II ERROR
C          6. KI      -  # OF REPLICATIONS
C          7. IARG    -  ARGUEMENT FOR URN RANDUM NUMBER GEN.
C          8. II      - NUMBER OF APPROXIMATION ENTRIES
C
      IMPLICIT REAL (A-H, N-Z)
      READ(5, 1) ICASE
    1 FORMAT(I10)
      DO 14 K = 1, ICASE
      READ(5, 2)THETAO, THETA1, ALFA, BETA, KI, IARG, II
    2 FORMAT(4F10.0,3I10)
C
C COMPUTE APPROXIMATE A & B
C
      A =  (1-BETA)/ALFA
      B =  BETA/(1-ALFA)
C
      WRITE(6, 3)THETAO, THETA1, ALFA, BETA, A, B, KI
    3 FORMAT(1H1,9X,'TEST: NULL HYPOTHESIS: THETA = ',F7.3,/
     */10X,'TEST: ALTERNATE HYPOTHESIS: THETA = ',F7.3,//
     *10X,'TYPE  I ERROR = ',F5.3,'      PROB(REJECT NULL HYP'
     *,', GIVEN NULL HYP. TRUE)' ,//10X,'TYPE II ERROR = ',
     *F5.3,'     PROB(ACCEPT NULL HYP, GIVEN ALT. HYP. TRUE)'
     *,//10X,'ACCEPTANCE BOUNDRY, A = ',F9.4, //10X,
     *'REJECTION BOUNDRY,  B = ',F9.5,//10X,'TOTAL # OBS AT'
     *,' EACH TEST POINT = ',I5,//3X,'THETA',T11,'OPERATING'
     *,' CHARACTERISTIC VALUE',T46,'EXPECTED SAMPLE SIZE',
     *T70,'VARIANCE S S',/T12,'APPROX.',T22,'EXACT',T30,
     *'DIFFERENCE',T42,'APPROX.',T52,'EXACT',T60,
     *'DIFFERENCE',T74,'EXACT',//)
C
      DO 13 MM= 1, II
      READ (5,4) AEN, THETA, ALT
    4 FORMAT(F10.5,F10.5,F10.6)
      O = 0.0
      XL = 0.0
      SXLSQ = 0.0
      R = URN(O)
C
      DO 10 J = 1, KI
      L = 1
      X =0.0
C
C COMPUTE BINOMIAL RANDOM VARIATE
C
    5 M = 1
      DO 7 I = 1,M
      R = URN(IARG)
      IF(R - THETA)6,6,7
    6 X = X + 1
    7 CONTINUE
C
C COMPUTE NOT REJECT VALUE FOR BINOMIAL ASSUMPTION (ANB)
C
       NUM = ALOG(B)   + L*ALOG((1-THETAO)/(1-THETA1))
       DEN = ALOG(THETA1/THETAO) -ALOG((1-THETA1)/(1-THETAO))
       ANB = NUM/DEN
C
C COMPUTE REJECTION VALUE FOR BINOMIAL ASSUMPTION  (RNB)
C
       NUMR = ALOG(A) + L*ALOG((1-THETAO)/(1-THETA1))
       RNB = NUMR/DEN
```

```
      IF(X.LE.ANB)GO TO 9
      IF(X.GE.BNB)GO TO 8
      L = L + 1
      GO TO 5
    8 O = O + 1.
    9 XL = XL + L
      XLSQ = L ** 2
      SXLSQ = SXLSQ + XLSQ
   10 CONTINUE
      TALFA = 1 - O/KI
      XN = XL/KI
      XNXBSQ = KI   XN - XN
      VN = (SXLSQ - XNXBSQ)/(KI - 1)
      DAT = TALFA - ALT
      DAXN = XN - AEN
C
      WRITE(6,11)THETA, ALT, TALFA, DAT, AEN, XN, DAXN, VN
   11 FORMAT(2X,F6.4,T12,F7.4,T21,F6.4,T32,F6.4,T42,F7.4,T51
     *,F7.4,T61,F7.4,T73,F7.2,/)
      WRITE(7,12)THETA, ALT, TALFA, DAT, AEN, XN, DAXN, VN
   12 FORMAT(8F10.4)
C
   13 CONTINUE
   14 CONTINUE
      STOP
      END
```

COMPUTER PROGRAM -- EXACT NORMAL

```
C  INPUT VARIABLES ARE:
C
C             1. ICASE   - # OF CASES TO SIMULATE
C             2. THETA0  - NULL HYPOTHESIS (H) VARIABLE
C             3. THETA1  - ALTERNATE HYPOTHESIS (A) VARIABLE
C             4. ALFA    - TYPE I  ERROR
C             5. BETA    - TYPE II ERROR
C             6. STDX    -  STANDARD DEVIATION OF SAMPLE DIST'N.
C             7. KI      -  # OF REPLICATIONS
C             8. IARG    -  ARGUEMENT FOR URN RANDUM NUMBER GEN.
C             9. II      - NUMBER OF APPROXIMATION ENTRIES
C
      IMPLICIT REAL (A-H, N-Z)
      READ(5, 1) ICASE
    1 FORMAT(I10)
      DO 13 K=1, ICASE
      READ(5,2) THETA0,THETA1,ALFA, BETA, STDX, KI, IARG, II
    2 FORMAT(5F10.0,3I10)
C
C   COMPUTE APPROXIMATE A & B
C
      A = (1-BETA)/ALFA
      B = BETA/(1-ALFA)
C
      WRITE(6,3)THETA0,THETA1,ALFA, BETA, STDX,A,B,KI
    3 FORMAT(1H1,9X,'TEST: NULL HYPOTHESIS: THETA = ',F7.3,/
     */10X,'TEST: ALTERNATE HYPOTHESIS: THETA = ',F7.3,//
     *10X,'TYPE  I ERROR = ',F5.3,'     PROB(REJECT NULL HYP'
     *,', GIVEN NULL HYP. TRUE)' ,//10X,'TYPE II ERROR = ',
     *F5.3,'     PROB(ACCEPT NULL HYP. GIVEN ALT. HYP. TRUE)'
     *,//10X,'STANDARD DEVIATION = ', F5.1
     *,//10X,'ACCEPTANCE BOUNDRY, A = ',F9.4, //10X,
     *'REJECTION BOUNDRY,  B = ',F9.5,//10X,'TOTAL # OBS AT'
     *,' EACH TEST POINT = ',I5,//3X,'THETA',T11,'OPERATING'
     *,' CHARACTERISTIC VALUE',T46,'EXPECTED SAMPLE SIZE',
     *T70,'VARIANCE S S',/T12,'APPROX.',T22,'EXACT',T30,
     *'DIFFERENCE',T42,'APPROX.',T52,'EXACT',T60,
     *'DIFFERENCE',T74,'EXACT',//)
C
      DO 12 M=1,II
      READ(5,4) AEN, THETA, ALT
    4 FORMAT(F10.5,F10.5,F10.6)
      O=0.0
      XL = 0.0
      SXLSQ = 0.0
C
      R=URN(O)
      DO 9 J=1,KI
      L=1
      SUMX=0.0
    5 SUM = 0.0
C
C   COMPUTE NORMAL RANDOM VARIATE
C
      DO 6 I=1,12
      R = URN(IARG)
    6 SUM = SUM + R
      X = STDX * (SUM - 6.0) + THETA
      SUMX = SUMX + X
C
C   COMPUTE NOT REJECT VALUE FOR NORMAL ASSUMPTION - MEAN
C
      ANNM = ((STDX**2)/(THETA1-THETA0)) * ALOG(B) +
     * L*(THETA0 + THETA1)/2
C
C   COMPUTE REJECTION VALUE FOR NORMAL ASSUMPTION - MEAN
C
      RNNM = ((STDX**2)/(THETA1-THETA0))*ALOG(A) +
     * L*(THETA0 + THETA1)/2
```

```
      IF(SUMX.LE.ANNM) GO TO 8
      IF(SUMX.GE.BNNM)GO TO 7
      L=L+1
      GO TO 5
    7 Q = Q +1.
    8 XL = XL + L
      XLSQ = L ** 2
      SXLSQ = SXLSQ + XLSQ
    9 CONTINUE
      TALFA = 1 - Q/KI
      XN = XL/KI
      XNXBSQ = KI * XN * XN
      VN = (SXLSQ - XNXBSQ)/(KI - 1)
      DAT = TALFA - ALT
      DAXN = XN - AEN
C
      WRITE(6,10)THETA, ALT, TALFA, DAT, AEN, XN, DAXN, VN
   10 FORMAT(2X,F6.4,T12,F7.4,T21,F6.4,T32,F6.4,T42,F7.4,T51
     *,F7.4,T61,F7.4,T73,F7.2,/)
      WRITE(7,11)THETA, ALT, TALFA, DAT, AEN, XN, DAXN, VN
   11 FORMAT(8F10.4)
C
   12 CONTINUE
   13 CONTINUE
      STOP
      END
```

COMPUTER PROGRAM -- EXACT EXPONENTIAL

```
C       EXPONENTIAL DISTRIBUTION:    F(X, T) =   T * EXP(-T*X)
C
C INPUT VARIABLES ARE:
C
C             1. ICASE   - # OF CASES TO SIMULATE
C             2. THETA0  - NULL HYPOTHESIS (H) VARIABLE          (T0)
C             3. THETA1  - ALTERNATE HYPOTHESIS (A) VARIABLE (T1)
C             4. ALFA    - TYPE I   ERROR
C             5. BETA    - TYPE II ERROR
C             6. KI      -  # OF REPLICATIONS
C             7. IARG    -  ARGUEMENT FOR URN RANDUM NUMBER GEN.
C             8. II      - NUMBER OF APPROXIMATION ENTRIES
C
      IMPLICIT REAL (A-H, N-Z)
      READ(5, 1) ICASE
    1 FORMAT(I10)
      DO 12 K = 1, ICASE
      READ(5, 2)THETA0, THETA1, ALFA, BETA, KI, IARG, II
    2 FORMAT(4F10.0,3I10)
C
C COMPUTE APPROXIMATE A & B
C
      A = (1-BETA)/ALFA
      B = BETA/(1-ALFA)
C
      WRITE(6, 3)THETA0, THETA1, ALFA, BETA, A, B, KI
    3 FORMAT(1H1,9X,'TEST: NULL HYPOTHESIS: THETA = ',F7.3,/
     */10X,'TEST: ALTERNATE HYPOTHESIS: THETA = ',F7.3,//
     *10X,'TYPE  I ERROR = ',F5.3,'      PROB(REJECT NULL HYP'
     *,', GIVEN NULL HYP. TRUE)' ,//10X,'TYPE II ERROR = ',
     *F5.3,'      PROB(ACCEPT NULL HYP, GIVEN ALT. HYP. TRUE)'
     *,//10X,'ACCEPTANCE BOUNDRY, A = ',F9.4, //10X,
     *'REJECTION BOUNDRY,  B = ',F9.5,//10X,'TOTAL # OBS AT'
     *,' EACH TEST POINT = ',I5,//3X,'THETA',T11,'OPERATING'
     *,' CHARACTERISTIC VALUE',T46,'EXPECTED SAMPLE SIZE',
     *T70,'VARIANCE S S',/T12,'APPROX.',T22,'EXACT',T30,
     *'DIFFERENCE',T42,'APPROX.',T52,'EXACT',T60,
     *'DIFFERENCE',T74,'EXACT',//)
C
      DO 11 M = 1, II
      READ (5,4) AEN, THETA, ALT
    4 FORMAT(F10.5,F10.5,F10.6)
C
      O = 0.0
      XL = 0.0
      SXLSQ = 0.0
      R = URN(0)
C
      DO 8  J = 1, KI
      L = 1
      SUMX = 0.0
C
C COMPUTE EXPONENTIAL RANDOM VARIATE
C
    5 R = URN(IARG)
      X = (-1.0/THETA) * ALOG(R)
      SUMX = SUMX + X
C
C COMPUTE NOT REJECT VALUE FOR EXPONENTIAL ASSUMPTION (ANE)
C
      NUME = -ALOG(B) + L*ALOG(THETA1/THETA0)
      DENE = THETA1 - THETA0
      ANE = NUME/DENE
C
C COMPUTE REJECTION VALUE FOR EXPONENTIAL ASSUMPTION (RNE)
C
      NUMRE = -ALOG(A) + L*ALOG(THETA1/THETA0)
      RNE = NUMRE/DENE
```

```
      IF(SUMX.GE.ANF)GO TO 7
      IF(SUMX.LE.BNF)GO TO 6
      L = L + 1
      GO TO 5
    6 Q = Q + 1.
    7 XL = XL + L
      XLSQ = L ** 2
      SXLSQ = SXLSQ + XLSQ
    8 CONTINUE
      TALFA = 1 - Q/KI
      XN = XL/KI
      XNXBSQ = KI * XN * XN
      VN = (SXLSQ - XNXBSQ)/(KI - 1)
      DAT = TALFA - ALT
      DAXN = XN - AEN
C
      WRITE(6, 9)THETA, ALT, TALFA, DAT, AEN, XN, DAXN, VN
    9 FORMAT(2X,F6.4,T12,F7.4,T21,F6.4,T32,F6.4,T42,F7.4,T51
     *,F7.4,T61,F7.4,T73,F7.2,/)
      WRITE(7,10)THETA, ALT, TALFA, DAT, AEN, XN, DAXN, VN
   10 FORMAT(8F10.4)
C
   11 CONTINUE
   12 CONTINUE
      STOP
      END
```

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DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

ORIGINATING ACTIVITY (Corporate author) Naval Postgraduate School Monterey, California 93940	2a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED 2b. GROUP

REPORT TITLE

AN EXAMINATION OF THE WALD STOPPING BOUNDS FOR THE SEQUENTIAL PROBABILITY RATIO TEST

DESCRIPTIVE NOTES (Type of report and inclusive dates)

Master's Thesis; April 1970

AUTHOR(S) (First name, middle initial, last name)

Michael William Gavlak, Lieutenant Commander, United States Navy

REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
April 1970	95	68

CONTRACT OR GRANT NO. PROJECT NO	9a. ORIGINATOR'S REPORT NUMBER(S) 9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)

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ABSTRACT

An examination of the Wald stopping bounds for the Sequential Probability Ratio Test (SPRT) is made by comparing results obtained from Monte Carlo simulations of sequential sampling tests with results obtained using Wald formulations. Operating Characteristic, ASN, and V[N] values are presented for test sampling from each of eight Binomial, 14 Exponential, and 24 Normal distributions. An extensive bibliography of references associated with SPRT is included.

DD FORM 1 NOV 65 1473 (PAGE 1)
S/N 0101-807-6811

A-31408

14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
SEQUENTIAL PROBABILITY RATIO TEST						
WALD STOPPING BOUNDS						
STATISTICAL SAMPLING						
BINOMIAL SAMPLING PLAN						
NORMAL SAMPLING PLAN						
EXPONENTIAL SAMPLING PLAN						
MONTE CARLO SIMULATION						
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An examination of the Wald stopping bounds for the sequential probability ratio test.